STATE BOARD FOR EDUCATOR CERTIFICATION/TEXAS EDUCATION AGENCY

TEXAS EDUCATOR CERTIFICATION

TExES/ExCET TOPT TExMaT TASC/TASC-ASL

TExMaT | Texas Examinations for Master Teachers

Preparation Manual

089 Master Mathematics Teacher 8–12

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PREFACE

The Texas Examinations for Master Teachers (TExMaT) Program has its origins in legislation passed in 1999 (House Bill 2307) that required the creation of the Master Reading Teacher (MRT) Certificate, the development of standards for the certificate, and the development of a Master Reading Teacher examination. In 2001, the Texas legislature passed legislation creating two additional categories of Master Teacher Certificates, the Master Mathematics Teacher (three certificates: Early Childhood–Grade 4, Grades 4–8, and Grades 8–12) and Master Technology Teacher (Early Childhood–Grade 12).

The Master Mathematics Teacher Certificate was created by the 77th Texas Legislature "to ensure that there are teachers with special training to work with other teachers and with students in order to improve student mathematics performance. . . ." A Master Mathematics Teacher will be an individual who holds a Master Mathematics Teacher Certificate and whose primary duties are to teach mathematics and to serve as a mathematics teacher mentor to other teachers.

A Master Mathematics Teacher Certificate may be obtained by individuals who:

- hold a teaching certificate,
- have at least three years of teaching experience,
- complete an SBEC-approved Master Mathematics Teacher preparation program, AND
- pass the TExMaT Master Mathematics Teacher EC–4, 4–8, *or* 8–12 certification examination.

The development of the educator standards for the Master Mathematics Teacher Certificates was completed in November 2001. The first SBEC-approved Master Mathematics Teacher preparation programs became available during the summer of 2002. The TExMaT Master Mathematics Teacher certification examinations will be administered for the first time in the summer of 2003.

This manual is designed to help examinees prepare for the new Master Mathematics Teacher 8–12 test. Its purpose is to familiarize examinees with the competencies to be tested, test item formats, and pertinent study resources. Educator preparation program staff may also find this information useful as they help examinees prepare for careers as Texas Master Teachers.

More information about the new TExMaT tests and the educator standards can be found at http://www.sbec.state.tx.us.

KEY FEATURES OF THE MANUAL

List of competencies that will be tested

Strategies for answering test questions

Sample test items and answer key

If you have questions after reading this preparation manual, please contact the State Board for Educator Certification, Office of Accountability at 1-512-238-3200.

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SECTION I

THE NEW TEXMAT TESTS

Successful performance on the TExMaT examination is required for the issuance of a Texas Master Teacher certificate. Each TExMaT test is a criterion-referenced examination designed to measure the knowledge and skills delineated in the corresponding TExMaT test framework. Each test framework is based on standards that were developed by Texas educators and other education stakeholders.

Each TExMaT test is designed to measure the requisite knowledge and skills that an initially certified Texas Master Teacher in this field must possess. This test includes multiple-choice items (questions) as well as a case study assignment for which candidates will construct a written response.

Development of the New TExMaT Tests

Committees of Texas educators and interested citizens guide the development of the new TExMaT tests by participating in each stage of the test development process. These working committees are comprised of Texas educators from public and charter schools, faculty from educator preparation programs, education service center staff, representatives from professional educator organizations, content experts, and members of the business community. The committees are balanced in terms of position, affiliation, years of experience, ethnicity, gender, and geographical location. The committee membership is rotated during the development process so that numerous Texas stakeholders may be actively involved. The steps in the process to develop the TExMaT tests are described below.

1. **Develop Standards.** Committees are convened to recommend what an initially certified Master Teacher in this field should know and be able to do. To ensure vertical alignment of standards across the range of instructional levels, individuals with expertise in early childhood, elementary, middle, or high school education meet jointly to articulate the critical knowledge and skills for a particular content area. Participants begin their dialogue using a "clean slate" approach with the Texas Essential Knowledge and Skills (TEKS) as the focal point. Draft standards are written to incorporate the TEKS and to expand upon that content to ensure that an initially certified Master Teacher in this field possesses the appropriate level of both knowledge and skills to instruct successfully.
2. **Review Standards.** Committees review and revise the draft standards. The revised draft standards are then placed on the SBEC Web site for public review and comment. These comments are used to prepare a final draft of the standards that will be presented to the SBEC Board for discussion, the State Board of Education (SBOE) for review and comment, and the SBEC Board for approval.
3. **Develop Test Frameworks.** Committees review and revise draft test frameworks that are based on the standards. These frameworks outline the specific competencies to be measured on the new TExMaT tests. The TExMaT competencies represent the critical components of the standards that can be measured with either a paper-and-pencil-based or a computer-based examination, as appropriate. Draft frameworks are not finalized until after the standards are approved and the job analysis/content validation survey (see #4) is complete.

4. **Conduct Job Analysis/Content Validation Surveys.** A representative sample of Texas educators who practice in or prepare individuals for each of the fields for which a Master Teacher certificate has been proposed are surveyed to determine the relative job importance of each competency outlined in the test framework for that content area. Frameworks are revised as needed following an analysis of the survey responses.
5. **Develop and Review New Test Items.** The test contractor develops draft items (multiple-choice and case study assignments) that are designed to measure the competencies described in the test framework. Committees review the newly developed test items that have been written to reflect the competencies in the new test frameworks and may accept, revise, or reject test items. Committee members scrutinize the draft items for appropriateness of content and difficulty; clarity; match to the competencies; and potential ethnic, gender, and regional bias.
6. **Conduct Pilot Test of New Test Items.** All of the newly developed test items that have been deemed acceptable by the item review committees are then administered to an appropriate sample of candidates for certification.
7. **Review Pilot Test Data.** Pilot test results are reviewed to ensure that the test items are valid, reliable, and free from bias.
8. **Administer New TExMaT Tests.** New TExMaT tests are constructed to reflect the competencies, and the tests are administered to candidates for certification.
9. **Set Passing Standard.** A Standard Setting Committee convenes to review performance data from the initial administration of each new TExMaT test and to recommend a final passing standard for that test. SBEC considers this recommendation as it establishes a passing score on the test.

Taking the TExMaT Master Mathematics Teacher Test and Receiving Scores

Please refer to the current TExMaT registration bulletin for information on test dates, sites, fees, registration procedures, and policies.

You will be mailed a score report approximately four weeks after each test you take. The report will indicate whether you have passed the test and will include:

- a total test *scaled* score. Scaled scores are reported to allow for the comparison of scores on the same content-area test taken on different test administration dates. The total scaled score is not the percentage of items answered correctly and is not determined by averaging the number of questions answered correctly in each domain.
  - For all TExMaT tests, the score scale is 100–300 with a scaled score of 240 as the minimum passing score. This score represents the minimum level of competency required to be a Master Teacher in this field in Texas public schools.
- a holistic score for your response to the case study assignment.
- your performance in the major content domains of the test and in the specific content competencies of the test.
  - This information may be useful in identifying strengths and weaknesses in your content preparation and can be used for further study or for preparing to retake the test.
- information to help you understand the score scale and interpret your results.

You will <u>not</u> receive a score report if you are absent or choose to cancel your score.

Additionally, unofficial score report information will be posted on the Internet on the score report mailing date of each test administration. Information about receiving unofficial scores via the Internet and other score report topics may be found on the SBEC Web site at www.sbec.state.tx.us.

Educator Standards

Complete, approved educator standards are posted on the SBEC Web site at www.sbec.state.tx.us.

SECTION II

USING THE TEST FRAMEWORK

The Texas Examinations for Master Teachers (TExMaT) test measures the content and professional knowledge required of an initially certified Master Teacher in this field. This manual is designed to guide your preparation by helping you become familiar with the material to be covered on the test.

When preparing for this test, you should focus on the competencies and descriptive statements, which delineate the content that is eligible for testing. A portion of the content is represented in the sample items that are included in this manual. These test questions represent only a *sample* of items. Thus, your test preparation should focus on the complete content eligible for testing, as specified in the competencies and descriptive statements.

Organization of the TExMaT Test Framework

The test framework is based on the educator standards for this field.

The content covered by this test is organized into broad areas of content called domains. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of competencies. Each competency is composed of two major parts:

1. the *competency statement,* which broadly defines what an initially certified Master Teacher in this field should know and be able to do, and
2. the *descriptive statements,* which describe in greater detail the knowledge and skills eligible for testing.

The educator standards being assessed within each domain are listed for reference at the beginning of the test framework. These are then followed by a complete set of the framework's competencies and descriptive statements.

An example of a competency and its accompanying descriptive statements is provided on the next page.

Sample Competency and Descriptive Statements

Master Mathematics Teacher 8–12

Competency:

The Master Mathematics Teacher 8–12 understands the real number system and its structure, operations, algorithms, and representations.

Descriptive Statements:

The Master Mathematics Teacher:

- Understands the concepts of place value and decimal representations of real numbers.
- Understands the algebraic structure of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group).
- Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).
- Selects and uses appropriate representations of real numbers for particular situations.
- Uses a variety of models (e.g., concrete, pictorial, geometric, symbolic) to represent operations, algorithms, and real numbers.
- Uses real numbers to model and solve a variety of problems.
- Uses deductive reasoning to simplify and justify algebraic processes.
- Demonstrates how some problems that have no solution in the integer or rational number systems have solutions in the real number system.

Studying for the TExMaT Test

The following steps may be helpful in preparing for the TExMaT test.

1. Identify the information the test will cover by reading through the test competencies (see the following pages in this section). *Within each domain* of this TExMaT test, each competency will receive approximately equal coverage.

2. Read each competency with its descriptive statements in order to get a more specific idea of the knowledge you will be required to demonstrate on the test. You may wish to use this review of the competencies to set priorities for your study time.

3. Review the "Preparation Resources" section of this manual for possible resources to consult. Also, compile key materials from your preparation coursework that are aligned with the competencies.

4. Study this manual for approaches to taking the test.

5. When using resources, concentrate on the key ideas and important concepts that are discussed in the competencies and descriptive statements.

NOTE: This preparation manual is the only TExMaT test study material endorsed by SBEC for this field. Other preparation materials may not accurately reflect the content of the test or the policies and procedures of the TExMaT Program.

Domain I **Number Concepts: Content, Instruction, and Assessment (approximately 16% of the test)**

Standards Assessed:

Standard I: Number Concepts: The Master Mathematics Teacher understands and applies knowledge of numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and the vertical alignment of number concepts to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

Domain II **Patterns and Algebra: Content, Instruction, and Assessment (approximately 18% of the test)**

Standards Assessed:

Standard II: Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

Domain III **Precalculus and Calculus: Content, Instruction, and Assessment (approximately 16% of the test)**

Standards Assessed:

Standard II: Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard III: Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

Domain IV **Geometry and Measurement: Content, Instruction, and Assessment (approximately 18% of the test)**

Standards Assessed:

Standard III: Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

Domain V **Probability and Statistics: Content, Instruction, and Assessment (approximately 14% of the test)**

Standards Assessed:

Standard IV: Probability and Statistics: The Master Mathematics Teacher understands probability and statistics, their applications, and the vertical alignment of probability and statistics to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

Domain VI **Mathematical Processes, Perspectives, Mentoring, and Leadership (approximately 18% of the test)**

Standards Assessed:

Standard V: Mathematical Processes: The Master Mathematics Teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.

Standard IX: Mentoring and Leadership: The Master Mathematics Teacher facilitates appropriate standards-based mathematics instruction by communicating and collaborating with educational stake-holders; mentoring, coaching, exhibiting leadership, and consulting with colleagues; providing professional development opportunities for faculty; and making instructional decisions based on data and supported by evidence from research.

Standard X: Mathematical Perspectives: The Master Mathematics Teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics, and the evolving nature of mathematics and mathematical knowledge.

DOMAIN I—NUMBER CONCEPTS: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 001

The Master Mathematics Teacher 8–12 understands the real number system and its structure, operations, algorithms, and representations.

The Master Mathematics Teacher:

- Understands the concepts of place value and decimal representations of real numbers.
- Understands the algebraic structure of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group).
- Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).
- Selects and uses appropriate representations of real numbers for particular situations.
- Uses a variety of models (e.g., concrete, pictorial, geometric, symbolic) to represent operations, algorithms, and real numbers.
- Uses real numbers to model and solve a variety of problems.
- Uses deductive reasoning to simplify and justify algebraic processes.
- Demonstrates how some problems that have no solution in the integer or rational number systems have solutions in the real number system.

Competency 002

The Master Mathematics Teacher 8–12 understands the complex number system and its structure, operations, algorithms, and representations.

The Master Mathematics Teacher:

- Understands the algebraic structure of the complex number system and its subsets (e.g., complex numbers as a field, complex addition as vector addition).
- Understands the properties and operations of complex numbers (e.g., complex conjugate, magnitude/modulus, multiplicative inverse).
- Describes and analyzes properties of subsets of the complex numbers (e.g., closure, identities).
- Selects and uses appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations.
- Describes complex number operations (e.g., addition, multiplication, powers, roots) using symbolic and geometric representations.
- Demonstrates how some problems that have no solution in the real number system have solutions in the complex number system.

Competency 003

The Master Mathematics Teacher 8–12 understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.

The Master Mathematics Teacher:

- Applies ideas from number theory (e.g., prime numbers and factorization, the Euclidean algorithm, divisibility, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) to solve problems.
- Applies number theory concepts and principles to justify and prove number relationships.
- Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, noncommutative operations).
- Uses a variety of manipulatives to represent number properties.
- Uses properties of numbers (e.g., fractions, decimals, percents, ratios, proportions) to model and solve real-world problems.
- Applies counting techniques, such as permutations and combinations, to quantify situations and solve problems.
- Uses estimation techniques to solve problems and evaluates the reasonableness of solutions.

Competency 004

The Master Mathematics Teacher 8–12 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop number concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use number concepts.
- Recognizes and uses the vertical alignment of number concepts across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of number concepts.
- Uses evidence of students' current understanding of number concepts to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within number concepts.
- Designs challenging and engaging problem-solving tasks that develop number-concepts content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates number concepts within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of number concepts.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 005

The Master Mathematics Teacher 8–12 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of number concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve number concepts, skills, and procedures.
- Knows how to teach number concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of number concepts.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning number concepts.
- Evaluates and modifies instruction to improve learning of number concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN II—PATTERNS AND ALGEBRA: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 006

The Master Mathematics Teacher 8–12 uses patterns to model and solve problems and formulate conjectures.

The Master Mathematics Teacher:

- Recognizes, extends, and generalizes patterns and relationships in information presented in a variety of ways using concrete models, geometric figures, tables, graphs, and algebraic expressions.
- Uses methods of recursion and iteration to model and solve problems.
- Uses the principle of mathematical induction.
- Analyzes the properties of sequences and series (e.g., Fibonacci, arithmetic, geometric) and uses them to solve problems involving finite and infinite processes.
- Understands how sequences and series are applied to solve problems in the mathematics of finance (e.g., simple, compound, and continuous interest rates; annuities).

Competency 007

The Master Mathematics Teacher 8–12 understands attributes of functions, relations, and their graphs.

The Master Mathematics Teacher:

- Understands when a relation is a function.
- Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.
- Understands that there exist functions that cannot be represented by any mathematical equation.
- Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).
- Identifies and analyzes even and odd functions, one-to-one functions, inverse functions, and their graphs.
- Applies basic transformations [e.g., $k\mathrm{f}(x)$, $\mathrm{f}(x) + k$, $\mathrm{f}(x - k)$, $\mathrm{f}(kx)$, $|\mathrm{f}(x)|$] to a parent function, *f,* and describes the effects on the graph of $y = \mathrm{f}(x)$.
- Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations, and describes the results using multiple representations (e.g., concrete, verbal, graphic, symbolic).
- Uses graphs of functions to formulate conjectures of identities [e.g., $y = x^2 - 1$ and $y = (x - 1)(x + 1)$, $y = \log x^3$ and $y = 3 \log x$, $y = \sin(x + \frac{\pi}{2})$ and $y = \cos x$].
- Represents and solves problems using parametric and polar equations using a variety of methods, including technology.

Competency 008

The Master Mathematics Teacher 8–12 understands linear functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.

The Master Mathematics Teacher:

- Understands the relationship between linear models and rate of change.
- Interprets the meaning of slope and intercept in a variety of situations.
- Analyzes the relationship between a linear equation and its graph.
- Writes equations of lines given various characteristics (e.g., two points, a point and slope, slope and y-intercept).
- Determines the linear function that best models a set of data.
- Uses a variety of methods (e.g., numeric, algebraic, graphic) to solve problems involving systems of linear equations and inequalities.
- Applies techniques of linear and matrix algebra to represent and solve problems involving linear systems.
- Models and solves problems involving linear equations and inequalities using a variety of methods, including technology.

Competency 009

The Master Mathematics Teacher 8–12 understands quadratic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.

The Master Mathematics Teacher:

- Manipulates and simplifies quadratic expressions.
- Analyzes the zeros (real and complex) of quadratic functions.
- Understands connections and translates among geometric, graphic, numeric, and symbolic representations of quadratic functions.
- Makes connections between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ representations of a quadratic function and its graph.
- Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).
- Models and solves problems involving quadratic equations, inequalities, and systems using a variety of methods, including technology.

Competency 010

The Master Mathematics Teacher 8–12 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop patterns and algebra concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use patterns and algebra.
- Recognizes and uses the vertical alignment of patterns and algebra across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that students develop accuracy and fluency of patterns and algebra.
- Uses evidence of students' current understanding of patterns and algebra to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within patterns and algebra.
- Designs challenging and engaging problem-solving tasks that develop patterns and algebra content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates patterns and algebra concepts within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of patterns and algebra.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 011

The Master Mathematics Teacher 8–12 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of patterns and algebra concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve patterns and algebra concepts, skills, and procedures.
- Knows how to teach patterns and algebra concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of patterns and algebra concepts.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning patterns and algebra concepts.
- Evaluates and modifies instruction to improve learning of patterns and algebra concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN III—PRECALCULUS AND CALCULUS: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 012

The Master Mathematics Teacher 8–12 understands polynomial, rational, radical, absolute value, and piecewise-defined functions and relations, analyzes their algebraic and graphical properties, and uses them to model and solve problems.

The Master Mathematics Teacher:

- Recognizes and translates among multiple representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value, and piecewise-defined functions and relations.
- Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value, and piecewise-defined functions and relations.
- Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or not differentiable) of a function, the graph of the function, and the function's symbolic representation.
- Analyzes functions and relations in terms of vertical, horizontal, and slant asymptotes.
- Analyzes and applies the relationships among inverse variation and rational functions and relations.
- Solves equations and inequalities involving polynomial, rational, radical, absolute value, and piecewise-defined functions and relations using a variety of methods (e.g., tables, algebraic methods, graphs, using a graphing calculator) and evaluates the reasonableness of solutions.
- Models situations using polynomial, rational, radical, absolute value, and piecewise-defined functions and relations and solves problems using a variety of methods, including technology.
- Identifies and analyzes the relationships between the properties of conic sections (e.g., foci, axes of symmetry, asymptotes, eccentricity) and their appropriate second-degree equation.
- Explores and solves application problems involving conic sections.

Competency 013

The Master Mathematics Teacher 8–12 understands exponential and logarithmic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.

The Master Mathematics Teacher:

- Recognizes and translates among multiple representations (e.g., written, numerical, tabular, graphical, algebraic) of exponential and logarithmic functions.
- Recognizes and uses connections among significant characteristics (e.g., intercepts, asymptotes) of a function involving exponential or logarithmic expressions, the graph of the function, and the function's symbolic representation.
- Understands the relationship between exponential and logarithmic functions and uses the laws and properties of exponents and logarithms to simplify expressions and solve problems.
- Uses a variety of representations and techniques (e.g., numerical methods, tables, graphs, analytic techniques, graphing calculators) to solve equations, inequalities, and systems involving exponential and logarithmic functions.
- Models and solves problems involving exponential growth and decay.
- Uses logarithmic scales (e.g., Richter, decibel) to describe phenomena and solve problems.
- Uses exponential and logarithmic functions to model and solve problems, including problems involving the mathematics of finance (e.g., compound interest).
- Uses the exponential function to model situations and solve problems in which the rate of change of a quantity is proportional to the current amount of the quantity [i.e., $f'(x) = k f(x)$].
- Models and solves problems involving logistic functions.

Competency 014

The Master Mathematics Teacher 8–12 understands trigonometric and circular functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.

The Master Mathematics Teacher:

- Analyzes the relationships among the unit circle in the coordinate plane, circular functions, and the trigonometric functions.
- Solves problems using angular- and linear-velocity concepts.
- Recognizes and translates among multiple representations (e.g., written, numerical, tabular, graphical, algebraic) of trigonometric functions and their inverses.
- Relates trigonometry and algebra by expressing equations in rectangular, parametric, and polar representations.
- Solves problems using the law of sines and the law of cosines.
- Recognizes and uses connections among significant properties (e.g., zeros, axes of symmetry, local extrema) and characteristics (e.g., amplitude, frequency, phase shift) of a trigonometric function, the graph of the function, and the function's symbolic representation.
- Understands the relationships between trigonometric functions and their inverses and uses these relationships to solve problems.
- Uses trigonometric identities to simplify expressions and solve equations.
- Models and solves a variety of problems (e.g., analyzing periodic phenomena) using trigonometric functions.
- Uses technology to analyze and solve problems involving trigonometric functions.

Competency 015

The Master Mathematics Teacher 8–12 understands and solves problems using limits, continuity, and differential calculus.

The Master Mathematics Teacher:

- Understands and applies the concepts of limit, continuity, and differentiability.
- Understands the definition and properties of the derivative.
- Relates the concept of average rate of change to the slope of the secant line and the concept of instantaneous rate of change to the slope of the tangent line.
- Understands and applies techniques of differentiation (e.g., product rule, chain rule).
- Uses the first and second derivatives to analyze the graph of a function (e.g., local extrema, concavity, points of inflection).
- Models and solves a variety of problems (e.g., velocity, acceleration, optimization, related rates) using differential calculus.
- Analyzes how technology can be used to solve problems and illustrate concepts involving differential calculus.

Competency 016

The Master Mathematics Teacher 8–12 understands and solves problems using integral calculus.

The Master Mathematics Teacher:

- Understands and applies the fundamental theorem of calculus and the relationship between differentiation and integration.
- Relates the concept of area under a curve to the limit of a Riemann sum.
- Understands and applies techniques of integration.
- Uses integral calculus to compute various measurements (e.g., area, volume, arc length) associated with curves and regions in the plane, and measurements associated with curves, surfaces, and regions in three-dimensional space.
- Models and solves a variety of problems (e.g., displacement, velocity, work, center of mass) using integral calculus.
- Analyzes how technology can be used to solve problems and illustrate concepts involving integral calculus.

Competency 017

The Master Mathematics Teacher 8–12 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop precalculus and calculus concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use precalculus and calculus.
- Recognizes and uses the vertical alignment of precalculus and calculus across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of precalculus and calculus.
- Uses evidence of students' current understanding of precalculus and calculus to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within precalculus and calculus.
- Designs challenging and engaging problem-solving tasks that develop precalculus and calculus content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates precalculus and calculus within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of precalculus and calculus.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 018

The Master Mathematics Teacher 8–12 implements a variety of instruction and assessment techniques to guide, evaluate, and improve student learning of precalculus and calculus concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve precalculus and calculus concepts, skills, and procedures.
- Knows how to teach precalculus and calculus concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of precalculus and calculus.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning precalculus and calculus.
- Evaluates and modifies instruction to improve learning of precalculus and calculus concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN IV—GEOMETRY AND MEASUREMENT: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 019

The Master Mathematics Teacher 8–12 understands measurement as a process.

The Master Mathematics Teacher:

- Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.
- Applies formulas for perimeter, area, surface area, and volume of geometric shapes and solids (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.
- Solves problems involving capacity, mass, weight, density, time, temperature, angles, and rates of change.
- Recognizes the effects on length, area, or volume when linear dimensions are changed.
- Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve measurement problems.
- Uses methods of approximation and estimation and understands the effects of error on measurement.

Competency 020

The Master Mathematics Teacher 8–12 understands geometries, in particular Euclidean geometry, as axiomatic systems.

The Master Mathematics Teacher:

- Understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
- Uses properties of points, lines, planes, angles, lengths, and distances to solve problems.
- Applies the properties of parallel and perpendicular lines to solve problems.
- Uses properties of congruence and similarity to explore geometric relationships, justify conjectures, and prove theorems.
- Describes and justifies geometric constructions made using compass and straightedge, reflection devices, and other appropriate technologies.
- Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.
- Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry.
- Demonstrates an understanding of proof, including indirect proof, in geometry, and provides convincing arguments or proofs for geometric theorems.

Competency 021

The Master Mathematics Teacher 8–12 understands the results, uses, and applications of Euclidean geometry.

The Master Mathematics Teacher:

- Analyzes the properties of polygons and their components.
- Analyzes the properties of circles and the lines that intersect them.
- Uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures (e.g., relationships of sides, angles).
- Computes the perimeter, area, and volume of shapes and solids created by subdividing and combining other shapes and solids (e.g., arc length, area of sectors, volume of a hemisphere).
- Analyzes cross sections and nets of three-dimensional solids.
- Uses top, front, side, and corner views of three-dimensional solids to create complete representations and solve problems.
- Applies properties of two- and three-dimensional figures to solve problems in other disciplines and in everyday life.

Competency 022

The Master Mathematics Teacher 8–12 understands coordinate, transformational, and vector geometry and their connections.

The Master Mathematics Teacher:

- Identifies transformations (i.e., reflections, translations, rotations, dilations) and explores their properties.
- Uses the properties of transformations and their compositions to solve problems.
- Uses transformations to explore and describe reflectional, rotational, and translational symmetry.
- Applies transformations in the coordinate plane.
- Applies concepts and properties of slope, midpoint, parallelism, perpendicularity, and distance to explore properties of geometric figures and solve problems in the coordinate plane.
- Uses rectangular and polar coordinate geometry to derive and explore the equations, properties, and applications of conic sections, including degenerate conic sections.
- Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.
- Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

Competency 023

The Master Mathematics Teacher 8–12 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop geometry and measurement concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use geometry and measurement.
- Recognizes and uses the vertical alignment of geometry and measurement across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of geometry and measurement.
- Uses evidence of students' current understanding of geometry and measurement to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within geometry and measurement.
- Designs challenging and engaging problem-solving tasks that develop geometry and measurement content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates geometry and measurement within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of geometry and measurement.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 024

The Master Mathematics Teacher 8–12 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of geometry and measurement concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve geometry and measurement concepts, skills, and procedures.
- Knows how to teach geometry and measurement concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of geometry and measurement.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning geometry and measurement.
- Evaluates and modifies instruction to improve learning of geometry and measurement concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN V—PROBABILITY AND STATISTICS: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 025

The Master Mathematics Teacher 8–12 understands how to use appropriate graphical and numerical techniques, including the use of technology, to explore, analyze, and represent data.

The Master Mathematics Teacher:

- Selects and uses an appropriate measurement scale (i.e., nominal, ordinal, interval, and ratio) to answer research questions and analyze data.
- Organizes, displays, and interprets data in a variety of formats (e.g., tables, frequency distributions, scatterplots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).
- Applies concepts of center, spread, shape, and skewness to describe a data distribution.
- Understands measures of central tendency (i.e., mean, median, and mode) and dispersion (i.e., range, interquartile range, variance, and standard deviation).
- Applies linear transformations to convert data and describes the effects of linear transformations on measures of central tendency and dispersion.
- Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers, and measures of central tendency and dispersion.
- Supports arguments, makes predictions, and draws conclusions using summary statistics and graphs to analyze and interpret univariate data.
- Recognizes and uses appropriate graphical displays and descriptive statistics for categorical and numerical data.

Competency 026

The Master Mathematics Teacher 8–12 understands concepts and applications of probability.

The Master Mathematics Teacher:

- Understands how to explore concepts of probability through sampling, experiments, and simulations.
- Generates and uses probability models to represent situations.
- Uses the concepts and principles of probability to describe the outcomes of simple and compound events.
- Determines probabilities by constructing sample spaces to model situations.
- Solves a variety of probability problems using combinations and permutations.
- Solves a variety of probability problems using ratios of areas of geometric regions.
- Calculates probabilities using the axioms of probability and related theorems and concepts, such as the addition rule, multiplication rule, and conditional probability.
- Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, uniform, normal).
- Understands expected value, variance, and standard deviation of probability distributions (e.g., binomial, geometric, uniform, normal).

Competency 027

The Master Mathematics Teacher 8–12 understands the relationships among probability theory, sampling, and statistical inference, and how statistical inference is used in making and evaluating predictions.

The Master Mathematics Teacher:

- Applies knowledge of designing, conducting, analyzing, and interpreting statistical experiments to investigate real-world problems.
- Analyzes and interprets statistical information (e.g., the results of polls and surveys) and recognizes misleading as well as valid uses of statistics.
- Understands random samples and sample statistics (e.g., sample size, confidence intervals, biased or unbiased estimators).
- Makes inferences about a population using binomial, normal, and geometric distributions.
- Describes and analyzes bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis, correlation coefficients).
- Understands how to transform nonlinear data into linear form in order to apply linear regression techniques to develop exponential, logarithmic, and power regression models.
- Understands the law of large numbers and the central limit theorem and their connections to the process of statistical inference.
- Estimates parameters (e.g., population mean and variance) using point estimators (e.g., sample mean and variance).
- Understands principles of hypothesis testing.
- Analyzes categorical data (e.g., frequency tables, chi-square).

Competency 028

The Master Mathematics Teacher 8–12 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop probability and statistics concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use probability and statistics.
- Recognizes and uses the vertical alignment of probability and statistics across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of probability and statistics.
- Uses evidence of students' current understanding of probability and statistics to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within probability and statistics.
- Designs challenging and engaging problem-solving tasks that develop probability and statistics content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates probability and statistics within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of probability and statistics.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 029

The Master Mathematics Teacher 8–12 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of probability and statistics concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve probability and statistics concepts, skills, and procedures.
- Knows how to teach probability and statistics concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of probability and statistics.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning probability and statistics.
- Evaluates and modifies instruction to improve learning of probability and statistics concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN VI—MATHEMATICAL PROCESSES, PERSPECTIVES, MENTORING, AND LEADERSHIP

Competency 030

The Master Mathematics Teacher 8–12 understands and uses mathematical processes to reason mathematically and solve problems.

The Master Mathematics Teacher:

- Demonstrates an understanding of the use of logical reasoning to evaluate mathematical conjectures and justifications and to provide convincing arguments or proofs for mathematical theorems.
- Applies correct mathematical reasoning to derive valid conclusions from a set of premises, and recognizes examples of fallacious reasoning.
- Demonstrates an understanding of the use of inductive reasoning to make conjectures and deductive methods to evaluate the validity of conjectures.
- Applies knowledge of the use of formal and informal reasoning to explore, investigate, and justify mathematical ideas.
- Recognizes that a mathematical problem can be solved in a variety of ways and selects an appropriate strategy for a given problem.
- Evaluates the reasonableness of a solution to a given problem.
- Demonstrates an understanding of estimation and evaluates its appropriate uses.
- Uses physical and numerical models to represent a given problem or mathematical procedure.
- Recognizes that assumptions are made when solving problems; then identifies and evaluates those assumptions.
- Investigates and explores problems that have multiple solutions.
- Applies content knowledge to develop a mathematical model of a real-world situation; then analyzes and evaluates how well the model represents the situation.
- Develops and uses simulations as a tool to model and solve problems.

Competency 031

The Master Mathematics Teacher 8–12 understands mathematical connections, the structure of mathematics, the historical development of mathematics, and how to communicate mathematical ideas and concepts.

The Master Mathematics Teacher:

- Recognizes and uses multiple representations of a mathematical concept.
- Uses mathematics to model and solve problems in other disciplines.
- Uses the structure of mathematical systems and their properties (e.g., mappings, inverse operations) to make connections among mathematical concepts.
- Recognizes the impacts of technological advances on mathematics (e.g., numerical versus analytical solutions) and of mathematics on technology (binary arithmetic).
- Emphasizes the role of mathematics in various careers and professions (e.g., economics, engineering) and how technology (e.g., spreadsheets, statistical software) affects the use of mathematics in various careers.
- Knows and uses the history and evolution of mathematical concepts, procedures, and ideas (e.g., the development of non-Euclidean geometry).
- Recognizes the contributions that different cultures have made to the field of mathematics.
- Uses current and professional resources to plan and develop activities that provide cultural, historical, and technological instruction for the classroom and that connect society and mathematics.
- Expresses mathematical statements using developmentally appropriate language, standard English, mathematical language, and symbolic mathematics.
- Communicates mathematical ideas using a wide range of technological tools and a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).
- Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams, and animations to communicate mathematical information.
- Uses the language of mathematics as a precise means of expressing mathematical ideas.

Competency 032

The Master Mathematics Teacher 8–12 knows how to communicate and collaborate with educational stakeholders to facilitate implementation of appropriate, standards-based mathematics instruction.

The Master Mathematics Teacher:

- Knows the dual role of the Master Mathematics Teacher as teacher and mentor in the school community.
- Knows leadership, communication, and facilitation skills and strategies.
- Knows and applies principles, guidelines, and professional ethical standards regarding collegial and professional collaborations, including issues related to confidentiality.
- Understands the importance of collaborating with administrators, colleagues, parents/guardians, and other members of the school community to establish and implement the roles of the Master Mathematics Teacher and ensure effective ongoing communication.
- Knows strategies for communicating effectively with stakeholders, including other teachers, about using programs and instructional techniques that are based on established research that supports their effectiveness with a range of students, including students who are at-risk.
- Knows strategies for building trust and a spirit of collaboration with other members of the school community to effect positive change in the school mathematics program and mathematics instruction.
- Knows how to use leadership skills to ensure the effectiveness and ongoing improvement of the school mathematics program, encourage support for the program, and engage others in improving the program.
- Knows strategies for collaborating with members of the school community to evaluate, negotiate, and establish priorities regarding the mathematics program and to facilitate mentoring, professional development, and parent/guardian training.
- Knows strategies for conferring with students, colleagues, administrators, and parents/guardians to discuss mathematics-related issues.
- Knows strategies for collaborating with teachers, administrators, and others to identify professional development needs, generate support for professional development programs, and ensure provision of effective professional development opportunities.

Competency 033

The Master Mathematics Teacher 8–12 knows how to provide professional development through mentoring, coaching, and consultation with colleagues to facilitate implementation of appropriate, standards-based mathematics instruction, and makes instructional decisions supported by established research.

The Master Mathematics Teacher:

- Knows and applies skills and strategies for mentoring, coaching, and consultation in the development, implementation, and evaluation of an effective mathematics program.
- Knows learning processes and procedures for facilitating adult learning.
- Knows strategies for facilitating positive change in instructional practices through professional development, mentoring, coaching, and consultation.
- Knows models and features of effective professional development programs that promote sustained applications in classroom practice (e.g., modeling, coaching, follow-up).
- Knows differences between consultation and supervision.
- Knows how to use mentoring, coaching, and consultation to facilitate team building for promoting student development in mathematics.
- Knows how to select and use strategies for collaborating with colleagues to identify needs related to mathematics instruction.
- Knows strategies for collaborating effectively with colleagues with varying levels of skill and experience and/or diverse philosophical approaches to mathematics instruction to develop, implement, and monitor mathematics programs.
- Knows how to select and use strategies to maximize effectiveness as a Master Mathematics Teacher, such as applying principles of time management and engaging in continuous self-assessment.
- Knows sources for locating information about established research on mathematics learning and understands methods and criteria for reviewing research on mathematics learning.
- Knows how to critically examine established research on mathematics learning, analyzes its usefulness for addressing instructional needs, and applies appropriate procedures for translating research on mathematics learning into practice.

SECTION III

APPROACHES TO ANSWERING MULTIPLE-CHOICE ITEMS

The purpose of this section is to describe multiple-choice item formats that you will see on the TExMaT Master Mathematics Teacher (MMT) test and to suggest possible ways to approach thinking about and answering the multiple-choice items. However, these approaches are not intended to replace familiar test-taking strategies with which you are already comfortable and that work for you.

The Master Mathematics Teacher 8–12 test is designed to include 80 scorable multiple-choice items and approximately 10 nonscorable items. Your final scaled score will be based only on scorable items. The nonscorable multiple-choice items are pilot tested by including them in the test in order to collect information about how these questions will perform under actual testing conditions. Nonscorable test items are not considered in calculating your score, and they are not identified on the test.

All multiple-choice questions on this test are designed to assess your knowledge of the content described in the test framework. The multiple-choice questions assess your ability to recall factual information **and** to think critically about the information, analyze it, consider it carefully, compare it with other knowledge you have, or make a judgment about it.

When you are ready to answer a multiple-choice question, you must choose one of four *answer choices* labeled A, B, C, and D. Then you must mark your choice on a separate answer sheet.

In addition to the multiple-choice questions, the MMT test will include one case study assignment. Please see Section V: Case Study Assignment.

Calculators. If you want to use a calculator, you must bring your own calculator to the test administration. However, only the brands and models listed in the TExMaT registration bulletin may be used at the test. All calculators on the approved list are graphing calculators. Graphing calculators perform all the operations of typical scientific calculators. **Test administration staff will clear the memory of your calculator both before and after the test.**

NOTE: Some test questions for Master Mathematics Teacher 8–12 are designed to be solved with a graphing calculator. It is therefore strongly recommended that you bring a graphing calculator with you to the test site. Sharing of calculators will not be permitted.

The approved calculator brands and models are subject to change. If there is a change, examinees will be notified.

Calculators may be used for both the multiple-choice and case study sections of the test.

Definitions and Formulas. A set of definitions and formulas will be provided in your test booklet. A copy of those definitions and formulas is also provided in Section IV of this preparation manual.

Multiple-Choice Item Formats

You may see the following two types of multiple-choice questions on the test.

- — Single items
- — Items with stimulus material

You may have two or more items related to a single stimulus. This group of items is called a cluster. Following the last item of a clustered item set containing two or more items, you will see the graphic illustrated below.

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

This graphic is used to separate these clustered items related to specific stimulus material from other items that follow.

On the following pages, you will find descriptions of these commonly used item formats, along with suggested approaches for answering each type of item. In the actual testing situation, you may mark the test items and/or write in the margins of your test booklet, **but your final response must be indicated on the answer sheet provided.**

SINGLE ITEMS

In the single item format, a problem is presented as a direct question or an incomplete statement, and four answer choices appear below the question. The following question is an example of this type. It tests knowledge of Master Mathematics Teacher 8–12 competency 0017: *The Master Mathematics Teacher 8–12 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop precalculus and calculus concepts, skills, and procedures.*

After introducing a technique of integration, a teacher asks students to evaluate the following integrals and to determine a pattern in the application of the technique:

- $\int 4xe^{x^2}\, dx$
- $\int \frac{3x^2}{\sqrt{2x^3 + 4}}\, dx$
- $\int (10x^4 + 42x^2)(x^5 + 7x^3 - 4)^{100}\, dx$

What technique of integration has the teacher introduced, and what is the pattern mentioned?

A. integration by *u*-substitution, and the integral of *u* is a factor of the integrand

B. integration by *u*-substitution, and the derivative of *u* is a factor of the integrand

C. integration by parts, where the integrand is written as the product of *u* and *dv*, and *u* is the integral of *dv*

D. integration by parts, where the integrand is written as the product of *u* and *dv*, and *u* is the derivative of *dv*

Suggested Approach

Read the question carefully and critically. Think about what it is asking and the situation it is describing. Eliminate any obviously wrong answers, select the correct answer choice, and mark it on your answer sheet.

For many integrands that are products or quotients, if an expression *u* can be defined such that the derivative of *u* is one of the factors (or a multiple thereof) of the integrand, then the appropriate method of integration is *u*-substitution. The notion behind this method is that the substitution of the appropriate *u* (and *du*) results in an integrand, in terms of the variable *u*, that is considerably easier to integrate than the original integrand.

In the first example given, u would be selected to be x^2, the derivative of which is $2x$. A multiple of this derivative is one of the factors of the integrand, i.e., $4x$, which equals $2 \times 2x$. In the second example, u would be selected to be $2x^3 + 4$, the derivative of which is $6x^2$. A multiple of this derivative is one of the factors of the integrand, i.e., $3x^2$, which equals $\frac{1}{2} \times 6x^2$. In the third example, u would be selected to be $x^5 + 7x^3 - 4$, the derivative of which is $5x^4 + 21x^2$. A multiple of this derivative is one of the factors of the integrand, i.e., $10x^4 + 42x^2$, which equals $2 \times (5x^4 + 21x^2)$. Therefore, option B is the correct response.

Option A is correct to state that the method introduced is integration by u-substitution, but the pattern stated is incorrect.

Option C is incorrect to state that the method introduced is integration by parts. The most appropriate use of integration by parts is when the given integrand can be written as the product of two factors u and dv, such that dv is easy to integrate and $(v \times du)$ is at least as easy, or easier, to integrate than the original integrand.

Option D is incorrect to state that the method introduced is integration by parts.

ITEMS WITH STIMULUS MATERIAL

Some questions are preceded by stimulus material that relates to the item. Some types of stimulus material included on the test are reading passages, graphics, tables, or a combination of these. In such cases, you will generally be given information followed by an event to analyze, a problem to solve, or a decision to make.

One or more items may be related to a single stimulus. You can use several different approaches to answer these types of questions. Some commonly used approaches are listed below.

Strategy 1 Skim the stimulus material to understand its purpose, its arrangement, and/or its content. Then read the item and refer again to the stimulus material to verify the correct answer.

Strategy 2 Read the item *before* considering the stimulus material. The content of the item will help you identify the purpose of the stimulus material and locate the information you need to answer the question.

Strategy 3 Use a combination of both strategies; apply the "read the stimulus first" strategy with shorter, more familiar stimuli and the "read the item first" strategy with longer, more complex, or less familiar stimuli. You can experiment with the sample items in this manual and then use the strategy with which you are most comfortable when you take the actual test.

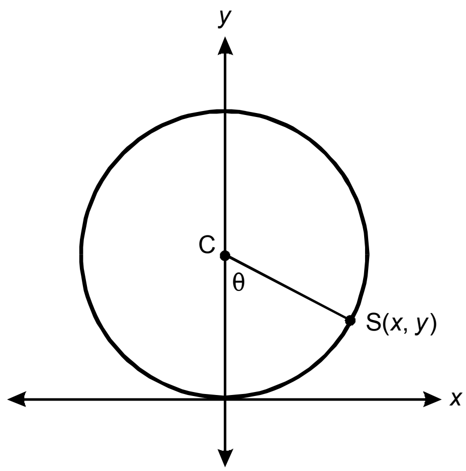
Whether you read the stimulus before or after you read the item, you should read it carefully and critically. You may want to underline its important points to help you answer the item.

As you consider items set in educational contexts, try to use the identified teacher's point of view to answer the items that accompany the stimulus. Be sure to consider the items in terms of only the information provided in the stimulus—not in terms of specific situations or individuals you may have encountered.

Suggested Approach

First read the stimulus (a diagram and description of a circular coin). A sample stimulus is shown below.

Use the diagram below to answer the two questions that follow.



A student marks a spot on the edge of a U.S. quarter and places it on its edge on a flat surface such that the spot touches the surface. The student then rolls the quarter. The diagram above represents a frontal view of the quarter on the surface, where *C* denotes the center of the circular coin, and *S* denotes the marked spot. A U.S. quarter has a diameter of approximately 2.35 centimeters.

Now you are prepared to address the first of the two questions associated with this stimulus. The first question measures Master Mathematics Teacher 8–12 competency 0014: *The Master Mathematics Teacher 8–12 understands trigonometric and circular functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.*

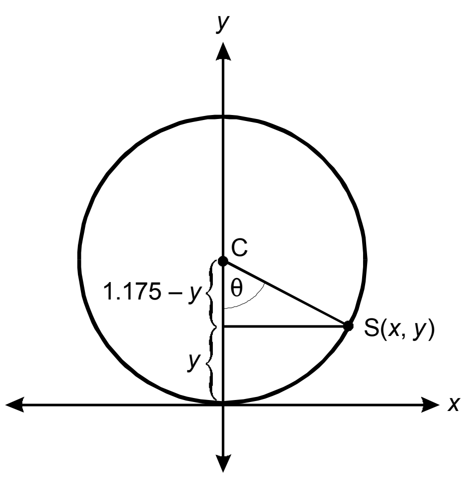
Which of the following best represents, in terms of θ, the distance between the spot on the edge of the quarter and the flat surface?

A. $y = -1.175 \cos \theta + 1.175$

B. $y = 1.175 \cos \theta$

C. $y = -2.35 \cos \theta + 2.35$

D. $y = 2.35 \cos \theta$



Consider that the diameter of the coin is 2.35 cm, so its radius is 1.175 cm. Connecting the point $S(x, y)$ to the y-axis with a horizontal line forms a right triangle with a height of $1.175 - y$ and a hypotenuse of 1.175 (the radius of the coin). You are asked to find y in terms of θ. From the right triangle formed, you know that $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{1.175 - y}{1.175}$. Multiplying both sides of the above equation by 1.175 gives $1.175 \cos \theta = 1.175 - y$. This results in $y = -1.175 \cos \theta + 1.175$. Therefore, option A is the correct response.

Option B results from using y as the height of the right triangle formed.

Option C results from using 2.35 cm as the radius of the coin.

Option D results from using y as the height of the right triangle formed and 2.35 cm as the radius of the coin.

Now you are ready to answer the next question. The second question measures competency 0014: *The Master Mathematics Teacher 8–12 understands trigonometric and circular functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.*

If the student rolls the coin in such a way that it makes a complete revolution in 0.3 seconds, what is the value of θ in terms of time t?

A. $\theta = \frac{3\pi}{5} t$

B. $\theta = \frac{3\pi}{10} t$

C. $\theta = \frac{10\pi}{3} t$

D. $\theta = \frac{20\pi}{3} t$

Consider carefully the information presented in the stimulus. Then read and reflect on the second question.

Consider that, for a complete revolution of the coin, θ equals 2π, and the coin makes a complete revolution in 0.3 seconds. Therefore, for any value of θ, the following relationship holds: $\frac{\theta}{2\pi} = \frac{t}{0.3}$. Multiplying both sides of this equation by 2π gives $\theta = \frac{2\pi t}{0.3} = \frac{2\pi t}{\frac{3}{10}} = \frac{20\pi t}{3}$. Therefore, option D is the correct response.

Option A results from using an incorrect relationship between θ and t, i.e., using $\frac{\theta}{2\pi} = 0.3t$.

Option B results from using an incorrect relationship between θ and t, i.e., using $\frac{\theta}{\pi} = 0.3t$.

Option C results from using π to represent the angle of one complete revolution.

SECTION IV

SAMPLE MULTIPLE-CHOICE ITEMS

This section presents some sample multiple-choice items for you to review as part of your preparation for the test. To demonstrate how each competency may be assessed, each sample item is accompanied by the competency number that it measures. While studying, you may wish to read the competency before and after you consider each sample item. Please note that the competency numbers will not appear on the actual test form.

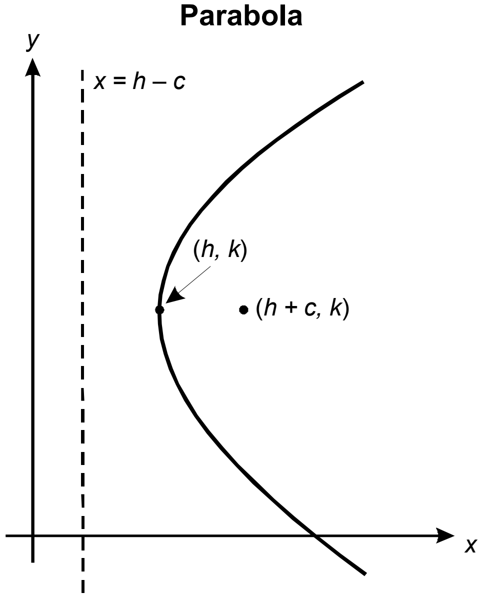
An answer key follows the sample items. The answer key lists the item number and correct answer for each sample item. Please note that the answer key also lists the competency assessed by each item and that the sample items are not necessarily presented in competency order.

The sample items are included to illustrate the formats and types of items you will see on the test; however, your performance on the sample items should not be viewed as a predictor of your performance on the actual examination.

Definitions and Formulas for Use on Mathematics Items

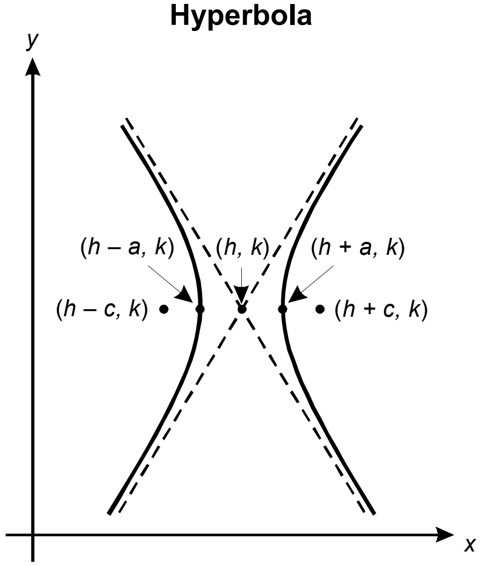
GEOMETRY

Parabola



$(y - k)^2 = 4c(x - h)$, where $c > 0$

Hyperbola



$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1,$$

where $b^2 = c^2 - a^2$

ALGEBRA

For $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $(a \neq 0)$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest,
where A is the final value
P is the principal
r is the interest rate
t is the term
n is divisions within the term

$[x] = n$ Greatest integer function,
where n is the integer such that $n \leq x < n + 1$

VOLUME

Cylinder: (area of base) × height

Cone: $\frac{1}{3}$ (area of base) × height

Sphere: $\frac{4}{3}\pi$ (radius)^3

Prism: (area of base) × height

AREA

Triangle: $\frac{1}{2}$ base × height

Rhombus: $\frac{1}{2}$ diagonal$_1$ × diagonal$_2$

Trapezoid: $\frac{1}{2}$ height (base$_1$ + base$_2$)

Sphere: 4π (radius)^2

Circle: π (radius)^2

Lateral surface area of cylinder:
2π (radius) × height

TRIGONOMETRY

Basic identities

$\sec \theta = \dfrac{1}{\cos \theta}$ $\csc \theta = \dfrac{1}{\sin \theta}$

$\cot \theta = \dfrac{1}{\tan \theta}$ $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$

$\sin^2 \theta + \cos^2 \theta = 1$

Addition formulas

$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$

$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$

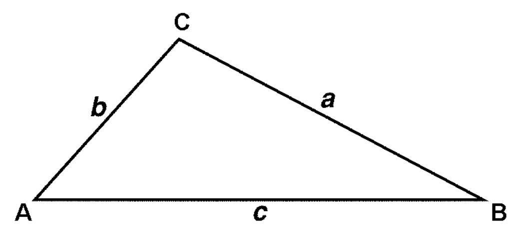
Law of sines $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Law of cosines

$c^2 = a^2 + b^2 - 2ab \cos C$

$b^2 = a^2 + c^2 - 2ac \cos B$

$a^2 = b^2 + c^2 - 2bc \cos A$



PROBABILITY & STATISTICS

Permutations: ${}_nP_k = \dfrac{n!}{(n-k)!}$

Combinations: ${}_nC_k = \dfrac{n!}{k!(n-k)!}$

Sample variance = $\dfrac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}$

Finite population variance = $\dfrac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$

END OF DEFINITIONS AND FORMULAS

Competency 001

1. Which of the following statements best describes the series $3 + \frac{3}{10} + \frac{3}{100} + \ldots$?

   A. arithmetic with a common difference of 10

   B. geometric with a common ratio of 10

   C. arithmetic with a common difference of $\frac{1}{10}$

   D. geometric with a common ratio of $\frac{1}{10}$

Competency 001

2. The endpoints of line segment $\overline{AB}$ are 0 and 1 and the endpoints of line segment $\overline{CD}$ are $\frac{1}{\sqrt{8}}$ and $\frac{1}{\sqrt{2}}$. What is the ratio of the length of $\overline{CD}$ to the length of $\overline{AB}$?

   A. $\frac{\sqrt{2}}{4}$

   B. $\frac{\sqrt{6}}{6}$

   C. $\frac{3\sqrt{2}}{8}$

   D. $\frac{\sqrt{2}}{2}$

Competency 002

3. If $z = 16(\cos 100° + i \sin 100°)$, which of the following represents $\sqrt{z}$?

   A. $4(\cos 10° + i \sin 10°)$

   B. $8(\cos 10° + i \sin 10°)$

   C. $4(\cos 50° + i \sin 50°)$

   D. $8(\cos 50° + i \sin 50°)$

Competency 003

4. A group of athletes consists of 4 swimmers, 12 runners, and 25 baseball players. From this group, how many ways can a team be formed consisting of 1 swimmer, 3 runners, and 6 baseball players?

   A. 177,324

   B. 155,848,000

   C. 1,121,099,408

   D. 673,263,360,000

Competency 003

5. On a trip from El Paso to Texarkana, a car is moving along a highway at a uniform rate of speed. At 9:00 A.M. it is $\frac{1}{4}$ of the way from El Paso to Texarkana. At 3:00 P.M. it is $\frac{7}{10}$ of the way from El Paso to Texarkana. Approximately what fraction of the way from El Paso to Texarkana was the car at 1:00 P.M.?

   A. $\frac{11}{20}$

   B. $\frac{3}{5}$

   C. $\frac{13}{20}$

   D. $\frac{7}{10}$

Competency 004

6. **Use the problem and Jane's response below to answer the question that follows.**

   > **<u>Problem:</u>**
   > A company manufactures 8 different men's colognes. If the company wants to prepare gift packages containing 5 different colognes, how many combinations of colognes are available?
   >
   > **<u>Jane's response:</u>**
   > $\frac{8!}{3!}$

   Which of the following is the best way for the teacher to respond to Jane?

   A. Do you know the definition of a factorial?

   B. Are you now able to simplify your answer?

   C. Do you want packages of 3 or 5 colognes?

   D. Is the order of the colognes important?

Competency 004

7. On a recent homework assignment, a student wrote that -4^2 is equivalent to 16. A review of which of the following concepts would best address the student's error?

   A. meaning of exponents

   B. order of operations

   C. multiplication of negative numbers

   D. distributive property

Competency 005

8. In a unit on matrices, a teacher wants to contrast the commutativity of integers under multiplication with the commutative property applied to matrices. To accomplish this task, she writes two distinct matrices, *A* and *B*, on the board and reminds students that "·" denotes matrix multiplication. In order to illustrate this contrast most effectively, which of the following assignments should the teacher give the students?

   A. Find $A \cdot A$ and $B \cdot B$.

   B. Find $6 \cdot A$ and $A \cdot 6$.

   C. Find $A \cdot B$ and $B \cdot A$.

   D. Find $A \cdot A^{-1}$ and $B \cdot B^{-1}$.

Competency 006

9. **Use the information below to answer the question that follows.**

	Number of Vertices	Number of Lateral Faces	Number of Edges
Triangular Prism	6	3	9
Rectangular Prism	8	4	12
Hexagonal Prism	12	6	18
Octagonal Prism	16	8	24

If the total number of vertices, lateral faces, and edges of a prism is 288, how many sides does the base of the prism have?

A. 42

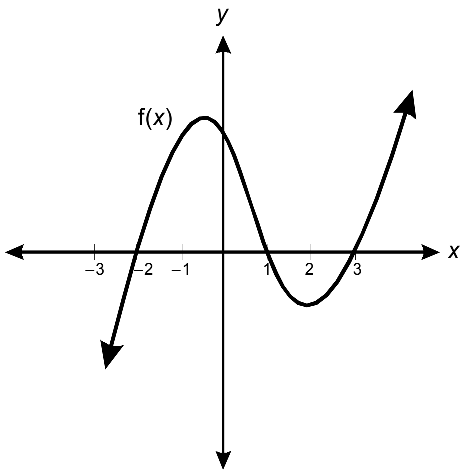
B. 44

C. 46

D. 48

Competency 007

10. **Use the graph below to answer the question that follows.**



If *f* is a cubic polynomial, on which of the following intervals can an inverse function for *f* be defined?

A. $(-\infty, 0)$

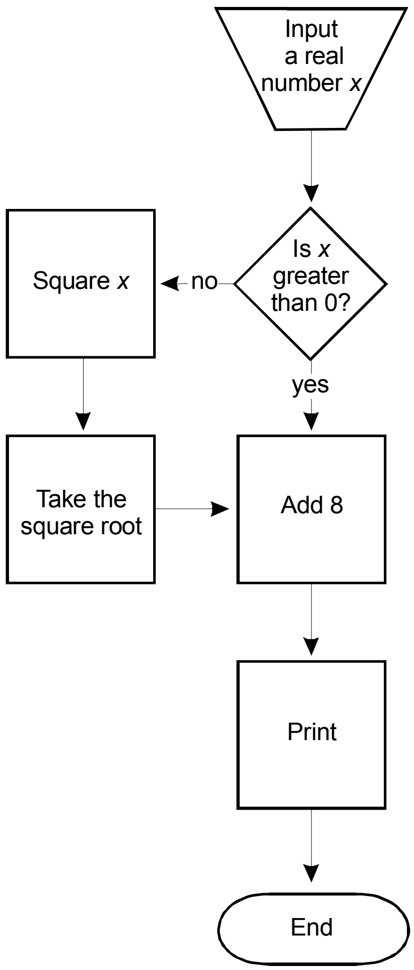
B. $(-2, 1)$

C. $(1, 3)$

D. $(2, \infty)$

Competency 007

11. **Use the flowchart below to answer the question that follows.**



What type of function does this flowchart represent?

A. linear

B. radical

C. quadratic

D. absolute value

Competency 008

12. What is the equation of the secant line through the graph of $y = \frac{1}{4}x^2 - 2x + 6$ at the points when $x = 6$ and $x = 8$?

    A. $3x - 2y - 6 = 0$

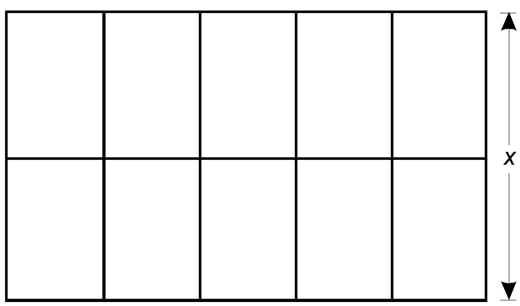
    B. $3x - 2y - 12 = 0$

    C. $2x - 3y + 2 = 0$

    D. $2x - 3y + 4 = 0$

Competency 009

13. **Use the diagram below to answer the question that follows.**



The diagram shows how a length of fence 180 feet long will be used to create 10 pens for holding animals. Which of the following equations represents the total area of the pens as a function of x?

A. $A(x) = 90x - x^2$

B. $A(x) = 60x - 2x^2$

C. $A(x) = 60x - 3x^2$

D. $A(x) = 60x - 6x^2$

Competency 009

14. **Use the information below to answer the question that follows.**

Number of Items Produced	Revenue (in dollars)
2	76
4	144
5	175
10	300
14	364
20	400

Students are told that the revenue of a firm, values of which are given above, is a function of the form $y = ax^2 + bx$. What is the value of b?

A. 36

B. 38

C. 40

D. 42

Competency 010

15. A student's solution to a homework problem involving systems of equations is shown below. A note from the student is included at the end of the problem.

Problem:

$$4x - 2y = 8$$
$$-2x + 3y = 20$$

Solution:

$$4x - 2y = 8$$
$$-2y = -4x + 8$$
$$y = 2x - 4$$

$$4x - 2(2x - 4) = 8$$
$$4x - 4x + 8 = 8$$
$$8 = 8$$

I'm not sure if I made an error. I solved the first equation for "y =", used substitution, and came up with the statement 8 = 8. I checked over all of my arithmetic. Did I do anything wrong?

Which of the following responses would best address the student's note?

A. There is nothing wrong with your approach. You've determined that the x value is 8. Now you just need to determine the y value.

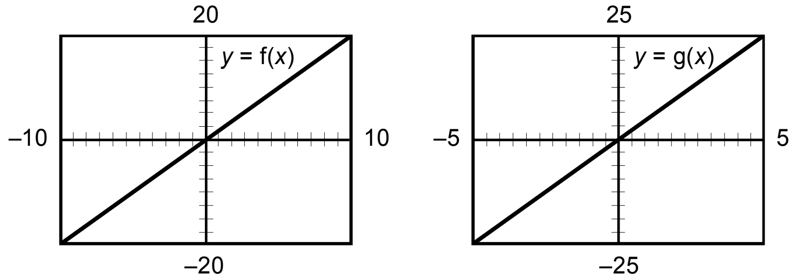
B. There is an error in your approach. You need to solve the first equation for "x =" and then use substitution to determine the y value.

C. There is nothing wrong with your approach. Your final statement of "8 = 8" indicates that there are an infinite number of solutions to the problem.

D. There is an error in your approach. After you solve the first equation for "y =", you need to substitute this expression for y into the other equation.

Competency 010

16. **Use the graphs below to answer the question that follows.**



A teacher asks the students in a class to use their graphing calculators to graph the functions $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, using the viewing windows shown above. This activity demonstrates that:

A. lines with different *y*-intercepts can appear to have the same *y*-intercept when viewed in different windows.

B. lines that appear to have the same slope in two viewing windows may have different slopes.

C. lines that have the same slope in two different viewing windows are parallel.

D. lines with the same *y*-intercept will appear equivalent when viewed in different windows.

Competency 011

17. **Use the class assignment below to answer the question that follows.**

Sketch the graph of each of the following: 1. $y = -7x^2 + 3$ 2. $y = x^2 - 16$ 3. $y = x^2 + 3x - 10$ 4. $y = -3x^2 + 12x + 15$

As students work on the assignment above, the teacher walks around the room and checks their progress. She observes that the majority of students have difficulty finding the x-intercepts. Before continuing with the topic on graphing, the teacher should assign problems of the form $y = ax^2 + bx + c$ to be:

A. solved for $x = 0$.

B. solved for $y = 0$.

C. expressed as $y = a(x - h)^2 + k$.

D. expressed as $\frac{1}{a}(y - k) = (x - h)^2$.

Competency 012

18. Which of the following points both satisfies the equation of an ellipse given by $x^2 + 4y^2 - 12x - 64 = 0$ and is farthest from the center of this ellipse?

A. $(-2, -3)$

B. $(-4, 0)$

C. $(6, 5)$

D. $(12, 4)$

Competency 013

19. The mass of the radioactive isotope carbon-14 changes with respect to time at a rate directly proportional to the mass of the isotope. If 100 grams of carbon-14 are present at time t = 0 years and 25 grams are left after 11,440 years, which of the following represents the constant of proportionality?

    A. $-\dfrac{\ln 4}{11440}$

    B. $-\dfrac{1}{2860}$

    C. $\dfrac{5}{2288}$

    D. $\dfrac{\ln 25}{11440}$

Competency 015

20. A spherical balloon is inflated at a rate of 3 in.^{3}/minute. How fast is the surface area of the balloon increasing when the radius is 14 inches?

    A. $\frac{3}{7}$ in.^{2}/minute

    B. $\frac{3}{784\pi}$ in.^{2}/minute

    C. $\frac{112}{3}$ in.^{2}/minute

    D. 112π in.^{2}/minute

Competency 016

21. What is the area bounded by the graph of $h(x) = -3x^2 + 3x + 6$ and the x-axis?

A. $\frac{13}{2}$

B. $\frac{17}{2}$

C. $\frac{23}{2}$

D. $\frac{27}{2}$

Competency 016

22. The definite integral $\int_1^2 (xy)\,dx$, for $x = 2\cos\theta$ and $y = 6\sin\theta$, is equivalent to which of the following?

A. $-12\int_1^2 (\cos\theta\sin\theta)\,dx$

B. $-12\int_{\frac{\pi}{3}}^1 (\cos\theta\sin\theta)\,d\theta$

C. $-24\int_{\frac{\pi}{6}}^0 (\cos\theta\sin^2\theta)\,d\theta$

D. $-24\int_{\frac{\pi}{3}}^0 (\cos\theta\sin^2\theta)\,d\theta$

Competency 018

23. **Use the dialogue below to answer the question that follows.**

Teacher:	Can you express sin(x + y) as the sum of two terms?
Student:	Yes, I get sin x + sin y.
Teacher:	How did you get your answer?
Student:	I used the distributive property.

Which of the following assignments would most effectively enable the student to realize that he is incorrect?

A. Graph sin x, sin y, and sin(x + y) on the same Cartesian coordinate system.

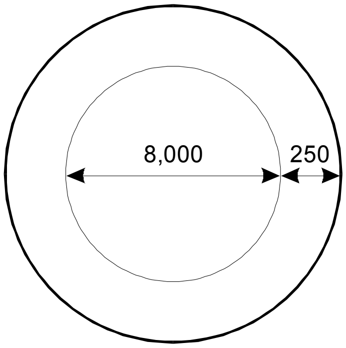
B. Determine the equivalence of sin(x + y) and sin x + sin y for $x = \frac{\pi}{2}$ and $y = \pi$.

C. Prove that the distributive property can only be applied to a nontrigonometric function.

D. Show that sin(x + y) = sin x cos y + sin y cos x for any values of x and y.

Competency 019

24. **Use the diagram below to answer the question that follows.**



The orbit of a satellite is approximately 250 miles above the surface of the earth, which has a diameter of approximately 8,000 miles. If the satellite travels a total of about 4,000,000 miles in space during a 240-hour period, it will orbit the earth approximately once every:

A. 90 minutes.

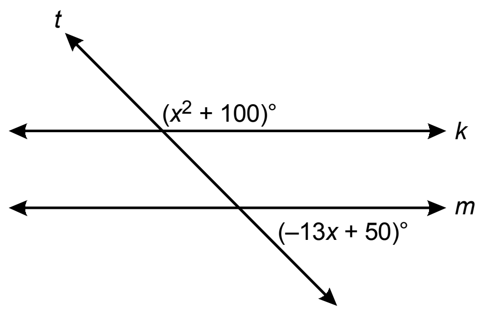
B. 96 minutes.

C. 180 minutes.

D. 192 minutes.

Competency 020

25. **Use the diagram below to answer the question that follows.**



Line *t* is a transversal for line *k* and line *m*. Which of the following could be solved to find value(s) of x such that *k* and *m* are parallel?

A. $x^2 – 13x – 30 = 0$

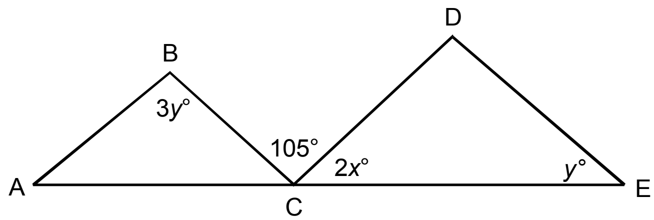
B. $x^2 – 13x – 60 = 0$

C. $x^2 + 13x + 50 = 0$

D. $x^2 + 13x – 50 = 0$

Competency 020

26. **Use the diagram below to answer the question that follows.**



If $\overline{AB}$ is parallel to $\overline{CD}$ and $\overline{BC}$ is parallel to $\overline{DE}$, what is the value of $x - y$?

A. -15

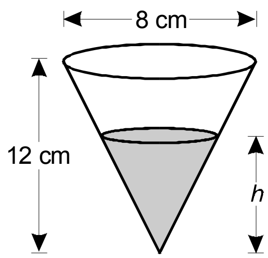
B. $-16\frac{1}{4}$

C. 15

D. $17\frac{1}{2}$

Competency 021

27. **Use the diagram below to answer the question that follows.**



The funnel above is a right circular cone with a diameter of 8 cm and a height of 12 cm. Which expression below represents the volume of fluid in the funnel in terms of *h*, the height of the fluid in the funnel?

A. $\frac{1}{27}\pi h^3$

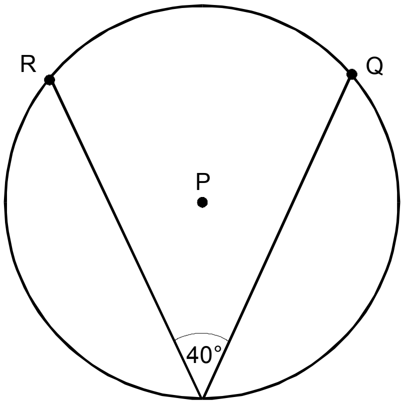
B. $\frac{2}{27}\pi h^3$

C. $\frac{1}{9}\pi h^3$

D. $\frac{2}{9}\pi h^3$

Competency 021

28. **Use the diagram below to answer the question that follows.**



The circle with center *P* shown above has a radius of 2 m. What is the length of $\overset{\frown}{RQ}$?

A. $\frac{2}{9}\pi$ m

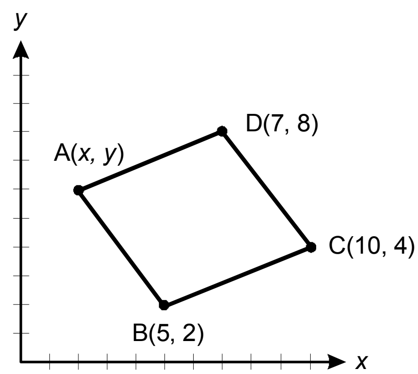
B. $\frac{4}{9}\pi$ m

C. $\frac{8}{9}\pi$ m

D. $\frac{32}{9}\pi$ m

Competency 022

29. **Use the diagram below to answer the question that follows.**



Quadrilateral *ABCD* is a parallelogram. What is an equation of line *AB*?

A. $3y - 4x = -23$

B. $3y - 4x = -14$

C. $3y + 4x = 23$

D. $3y + 4x = 26$

Competency 022

30. What is the equation of a circle that passes through the origin, has a radius of 12, and is centered in the second quadrant at the point $(-11, b)$?

A. $x^2 + y^2 + 22x - 2\sqrt{23}\,y = 0$

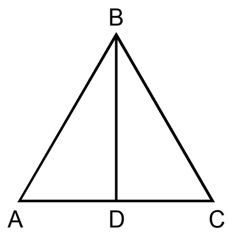
B. $x^2 + y^2 + 22x - 2\sqrt{23}\,y + 144 = 0$

C. $x^2 + y^2 + 22x - 24y + 121 = 0$

D. $x^2 + y^2 + 22x - 24y + 265 = 0$

Competency 023

31. **Use the figure and information below to answer the question that follows.**



In the figure of $\triangle ABC$ above, $AB = BC$ and $\overline{BD}$ bisects $\angle B$. Students are asked to prove that $\triangle ADB \cong \triangle CDB$. One student submits the following proof.

Statements	Reasons
1. $AB = BC$	1. Given
2. $AD = DC$	2. Definition of a bisector
3. $BD = BD$	3. Reflexive property
4. $\triangle ADB \cong \triangle CDB$	4. SSS

Which of the following best describes the error made by the student?

A. Reason 1 is an assumption and not a given.

B. Statement 2 assumes that the line segment is bisected.

C. Reason 3 should read *Symmetric property*.

D. Statement 4 does not follow from the previous statements.

Competency 024

32. Students are using a software program to investigate the properties of quadrilaterals. One student conjectures that if corresponding sides of two parallelograms are congruent, then the parallelograms are congruent. Which of the following questions would be most appropriate to challenge this student's mathematical thinking?

    A. Can you develop a two-column proof to demonstrate this conjecture?

    B. Can you find a counterexample to this conjecture?

    C. Can you explain how you would prove this conjecture using an indirect proof?

    D. Can you generalize this conjecture for all quadrilaterals?

Competency 025

33. For the set of numbers {20, 10, 55, 15, 30, 50, *x*}, the mean, median, and mode are all equal. What is the value of *x*?

    A. 15

    B. 20

    C. 30

    D. 50

Competency 026

34. Two tickets are drawn with replacement from a box containing four tickets numbered 1 through 4. What is the probability that the product of the numbers on the two tickets is 9 or greater?

    A. $\frac{1}{4}$

    B. $\frac{1}{3}$

    C. $\frac{1}{2}$

    D. $\frac{3}{4}$

Competency 027

35. The average content of a random sample of 900 gallon-containers of Healthy Farm milk is 1 gallon with a standard deviation of 0.05 gallons. Based on the results of this sample, which of the following provides the best estimate of the population of Healthy Farm gallon containers of milk?

    A. The average content is 1 gallon with a standard deviation of 0.05 gallons.

    B. The average content is 1 gallon with a standard deviation of 0.0017 gallons.

    C. The average content is 1.05 gallons with a standard deviation of 0.05 gallons.

    D. The average content is 1.05 gallons with a standard deviation of 0.0017 gallons.

Competency 027

36. A survey based on a random sample of 200 male giraffes evaluates a 95% confidence interval for the giraffes' average height to be 17' 10" ± 3". This statistic implies that:

    A. 95% of all giraffes are between 17' 7" and 18' 1" in height.

    B. 95% of all male giraffes are between 17' 7" and 18' 1" in height.

    C. there is a 95% chance that the interval between 17' 7" and 18' 1" includes the true average height of all giraffes.

    D. there is a 95% chance that the interval between 17' 7" and 18' 1" includes the true average height of all male giraffes.

Competency 028

37. Students are learning in their science class that biological diversity results from a genetic code that involves the arrangement of four different molecules into specific sequences. This topic would provide an opportunity in a mathematics class to discuss which of the following topics?

    A. probability

    B. exponential functions

    C. geometric progressions

    D. combinations and permutations

Competency 028

38. A teacher divides the class into groups of three students and gives each group a bag of coins. Each bag contains 15 dimes, 10 quarters, and 8 nickels. The class is asked to determine the following probabilities, assuming two draws are made without replacement from the bag of coins:

    P(2nd coin is a quarter given the 1st is a nickel)

    P(2nd coin is a dime given the 1st is a dime)

    P(2nd coin is a nickel given the 1st is a dime)

    Which of the following types of probability events is the class being taught?

    A. mutually exclusive

    B. complementary

    C. independent

    D. dependent

Competency 029

39. Students graph, on a scatterplot, a set of data on the ages and heights of a group of individuals and observe an approximately linear relationship. Which of the following computing tools could the students use to determine the equation of the line that best fits the data?

    A. regression models

    B. parametric graphs

    C. roots of a function

    D. numerical integration

Competency 030

40. An object placed in a tank of water for 24 hours has a temperature modeled by $H(t) = t(t + 2)(t - 15)(t - 30)$, where t is the number of hours in the tank. At what times, t, is the temperature of the object equal to zero?

    A. –2, 0, 15, and 30

    B. 0, 15, and 30

    C. 15 and 30

    D. 0 and 15

Competency 030

41. **Use the information below to answer the question that follows.**

$$\begin{aligned} \text{Let } x &= 3 \\ x^2 &= 3x \\ x^2 - 9 &= 3x - 9 \\ (x + 3)(x - 3) &= 3(x - 3) \\ x + 3 &= 3 \\ 6 &= 3 \end{aligned}$$

Which of the following is the reason that the fallacy above results?

A. squaring one side of an equation

B. improper substitution

C. division by zero

D. incorrect order of operations

Competency 031

42. The loudness of a sound, or sound intensity, (in watts per square meter) is inversely proportional to the square of the listener's distance from the source of the sound. At a distance of two meters, the sound intensity of a machine is 8.0×10^{-4}. What is the intensity at a distance of 10 meters from the machine?

    A. 3.2×10^{-5}

    B. 4.0×10^{-4}

    C. 1.6×10^{-3}

    D. 2.8×10^{-2}

Competency 031

43. **Use the information below to answer the question that follows.**

    1. Archimedes' *Measurement of the Circle*
    2. Descartes's *La Géométrie*
    3. Al-Khwarizmi's *Algebra*

    Which of the following sequences of lines represents the chronological order, from earliest to latest, in which the mathematical works listed above were written?

    A. 1, 2, 3

    B. 1, 3, 2

    C. 3, 1, 2

    D. 3, 2, 1

Competency 032

44. A new, schoolwide plan for improving mathematics instruction has been introduced at a local high school. The school principal asks the Master Mathematics Teacher to assess the ability of the school's mathematics teachers to meet the new instructional standards. The Master Mathematics Teacher determines that many of the teachers require further knowledge and skills in order to teach to the new standards effectively. Which of the following would be the most appropriate action for the Master Mathematics Teacher to take at this point?

    A. Survey teachers informally to determine whether they are interested in improving their knowledge and teaching skills.

    B. Ask teachers who possess the necessary knowledge and skills to suggest methods for assisting other teachers to implement the new methods.

    C. Revise the proposed instructional changes to a level consistent with the general knowledge and skills of the teachers.

    D. Meet with the principal to discuss providing professional development to equip teachers with the necessary knowledge and skills.

Competency 032

45. A faculty committee led by a school's Master Mathematics Teacher has recommended that the school purchase a group license for a mathematics software package. The Master Mathematics Teacher is most likely to gain administrative support for this purchase by:

    A. demonstrating the features and capabilities of the software to the administration.

    B. describing how the software will help mathematics teachers achieve the school's instructional goals.

    C. documenting the use of similar software at schools in surrounding districts.

    D. asking faculty and parents/guardians to voice their support for use of the software.

Competency 033

46. A teacher wants to adopt a problem-solving method that incorporates making a plan, carrying out the plan, and evaluating the solution of the problem for reasonableness. The educational writings of which of the following mathematicians would be the best resource for this teacher?

 A. Poincaré

 B. Hilbert

 C. Polya

 D. Fermat

Competency 033

47. A Master Mathematics Teacher observes a class in which the teacher introduces graphing concepts using mathematical terms with which many students are not familiar. As the class progresses, the students begin to talk among themselves and show a general lack of interest. The Master Mathematics Teacher recognizes that these students would benefit if the teacher initially used contextual language and then gradually transitioned into using mathematical terms. Which of the following is the best way for the Master Mathematics Teacher to make the classroom teacher aware of this need?

 A. Describe the students' behavior and ask the teacher reflective questions about its possible causes.

 B. Tell the teacher to introduce the graphing unit with a vocabulary session and provide students with copies of definitions of key mathematical terms.

 C. Give the teacher a list of terms students may not have understood and suggest using alternative terms.

 D. Provide the teacher with copies of research studies on the value of using real-life examples and everyday language for teaching graphing.

ANSWER KEY

Item Number	Correct Answer	Competency
1	**D**	001
2	**A**	001
3	**C**	002
4	**B**	003
5	**A**	003
6	**D**	004
7	**B**	004
8	**C**	005
9	**D**	006
10	**D**	007
11	**D**	007
12	**B**	008
13	**B**	009
14	**C**	009
15	**D**	010
16	**B**	010
17	**B**	011
18	**B**	012
19	**A**	013
20	**A**	015
21	**D**	016
22	**D**	016
23	**B**	018
24	**B**	019

Item Number	Correct Answer	Competency
25	**A**	020
26	**A**	020
27	**A**	021
28	**C**	021
29	**D**	022
30	**A**	022
31	**B**	023
32	**B**	024
33	**C**	025
34	**A**	026
35	**B**	027
36	**D**	027
37	**D**	028
38	**D**	028
39	**A**	029
40	**D**	030
41	**C**	030
42	**A**	031
43	**B**	031
44	**D**	032
45	**B**	032
46	**C**	033
47	**A**	033

SECTION V

CASE STUDY ASSIGNMENT

In addition to the multiple-choice section, the Master Mathematics Teacher (MMT) test will include one case study assignment that requires a written response. The written-response score will be combined with the multiple-choice score to produce a total test scaled score.

Included in this section is a description of the case study assignment, an explanation of the way case study assignment responses will be scored, and one sample case study assignment.

How Case Study Assignment Responses Are Scored

Responses will be scored on a four-point scale (see next page). Each point on the scale represents the degree to which the performance characteristics (see below) are demonstrated in the response.

The score point descriptions reflect typical responses at each score point. Although the score assigned corresponds to one of the score points, individual responses may include attributes of more than one score point.

PERFORMANCE CHARACTERISTICS

PURPOSE	The extent to which the candidate responds to the components of the assignment in relation to relevant competencies in the Master Mathematics Teacher 8–12 test framework.
APPLICATION OF KNOWLEDGE	Accuracy and effectiveness in the application of knowledge as described in relevant competencies in the Master Mathematics Teacher 8–12 test framework.
SUPPORT	Quality and relevance of supporting details in relation to relevant competencies in the Master Mathematics Teacher 8–12 test framework.
RATIONALE	Soundness of reasoning and depth of understanding of the assigned task in relation to relevant competencies in the Master Mathematics Teacher 8–12 test framework.
SYNTHESIS	The extent to which the candidate is able to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 8–12 in an applied context.

SCORE SCALE

Score	Score Point Description
4	**The "4" response reflects thorough knowledge and understanding of relevant competencies in the Master Mathematics Teacher 8–12 test framework.** • The response addresses all components of the assignment and fully completes the assigned task. • The response demonstrates an accurate and very effective application of relevant knowledge. • The response provides strong supporting evidence with specific and relevant examples. • The response demonstrates clear, logical reasoning and a comprehensive understanding of the assigned task. • The response demonstrates strong ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 8–12.
3	**The "3" response reflects sufficient knowledge and understanding of relevant competencies in the Master Mathematics Teacher 8–12 test framework.** • The response addresses most or all components of the assignment and sufficiently completes the assigned task. • The response demonstrates a generally accurate and effective application of relevant knowledge; minor problems in accuracy or effectiveness may be evident. • The response provides sufficient supporting evidence with mostly specific and relevant examples. • The response demonstrates sufficient reasoning and an overall understanding of the assigned task. • The response demonstrates sufficient ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 8–12.
2	**The "2" response reflects partial knowledge and understanding of relevant competencies in the Master Mathematics Teacher 8–12 test framework.** • The response addresses at least some components of the assignment and/or partially completes the assigned task. • The response demonstrates a partial and/or ineffective application of relevant knowledge; significant inaccuracies may be evident. • The response provides minimal supporting evidence with few relevant examples; some extraneous or unrelated information may be evident. • The response demonstrates limited reasoning and understanding of the assigned task. • The response demonstrates partial ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 8–12.
1	**The "1" response reflects little or no knowledge or understanding of relevant competencies in the Master Mathematics Teacher 8–12 test framework.** • The response addresses few components of the assignment and/or fails to complete the assigned task. • The response demonstrates a largely inaccurate and/or ineffective application of relevant knowledge. • The response provides little or no supporting evidence, few or no relevant examples, or many examples of extraneous or unrelated information. • The response demonstrates little or no reasoning or understanding of the assigned task. • The response demonstrates little or no ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 8–12.
U	The "U" (Unscorable) will be assigned to responses that are off topic/off task, illegible, primarily in a language other than English, or are too short or do not contain a sufficient amount of original work to score.
B	The "B" (Blank) will be assigned to written response booklets that are completely blank.

Note: Your written response should be your original work, written in your own words, and not copied or paraphrased from some other work.

Scoring Process

Each response will be evaluated according to the performance characteristics and assigned a holistic score on the score scale.

Case study assignment responses are scored on a scale of 1 to 4. Each response is evaluated by a minimum of two scorers with expertise in mathematics instruction. All scorers have successfully completed standardized orientation and are calibrated to the scoring criteria throughout the scoring session.

Analytic Notation

Examinees who do not pass the test and do not perform satisfactorily on the case study assignment will receive information concerning specific aspects of the written response that show a need for improvement. This information will be provided for examinees to use in preparing to retake the test.

If you do not pass the test or perform satisfactorily on the case study assignment, your score report will indicate one or more of the following areas for improvement in your written response. These areas are based on the performance characteristics in the score scale.

— Purpose

— Application of Knowledge

— Support

— Rationale

— Synthesis

Preparing for the Case Study Assignment

Following is one sample case study assignment that represents the type of question you will see on the MMT test.

In preparing for the case study assignment component of the test, you may wish to draft a response to the question by reading the case study and planning, writing, and revising your essay. You should plan to use about 90 minutes to respond to the sample case study assignment. Also, since no reference materials will be available during the test, it is recommended that you refrain from using a dictionary, a thesaurus, or textbooks while writing your practice response.

After you have written your practice response, review your response in light of the score point descriptions. You may also wish to review your response and the score scale with staff in your MMT preparation program.

General Directions for Responding to the Case Study Assignment

DIRECTIONS FOR CASE STUDY ASSIGNMENT
Master Mathematics Teacher 8–12

General Directions:

This section of the test consists of one case study assignment. For this assignment, you are to prepare a written response and record it in the area provided in the written response booklet.

Read the case study assignment carefully before you begin to write. Think about how you will organize what you plan to write. You may use any blank space provided in this test booklet to make notes, create an outline, or otherwise prepare your response. ***Your final response, however, must be written in the written response booklet.***

Evaluation Criteria:

Your written response will be evaluated based on the extent to which it demonstrates the knowledge and skills required to perform the roles of the Master Mathematics Teacher 8–12. You may draw from research and your professional experience. (Citing specific research is not required.)

Read the assignment carefully to ensure that you address all components. Your response to the assignment will be evaluated based on the following criteria:

- **PURPOSE:** The extent to which you respond to the components of the assignment in relation to relevant competencies in the Master Mathematics Teacher 8–12 test framework.
- **APPLICATION OF KNOWLEDGE:** Accuracy and effectiveness in the application of knowledge as described in relevant competencies in the Master Mathematics Teacher 8–12 test framework.
- **SUPPORT:** Quality and relevance of supporting details in relation to relevant competencies in the Master Mathematics Teacher 8–12 test framework.
- **RATIONALE:** Soundness of reasoning and depth of understanding of the assigned task in relation to relevant competencies in the Master Mathematics Teacher 8–12 test framework.
- **SYNTHESIS:** The extent to which you are able to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 8–12 in an applied context.

The assignment is intended to assess knowledge and skills required to perform the roles of the Master Mathematics Teacher 8–12, not writing ability. Your response, however, must be communicated clearly enough to permit a valid judgment about your knowledge and skills. Your response should be written for an audience of educators knowledgeable about the roles of the Master Mathematics Teacher 8–12.

The final version of your response should conform to the conventions of edited American English. Your response should be your original work, written in your own words, and not copied or paraphrased from some other work. You may, however, use citations when appropriate.

Sample Case Study Assignment

48. **Classroom Context:** This case study focuses on a ninth-grade mathematics teacher, Ms. Balmos, who is instructing her algebra students on a "new" kind of function—the exponential function. The class, which meets for 90 minutes every other day, is composed of students who achieve at various levels.

Master Mathematics Teacher Task: Ms. Balmos has asked the Master Mathematics Teacher (MMT) to observe her class and provide assistance teaching an introductory lesson on exponential functions. The MMT has agreed to observe her lesson. Ms. Balmos shows the MMT a lesson plan that she intends to use on the day of the MMT's observation. On the following pages, you will find:

- information from Ms. Balmos regarding previous instruction for this class;
- the lesson plan implemented on the day of the MMT's observation;
- an assignment given by Ms. Balmos to her class;
- excerpts of notes taken by the MMT while observing Ms. Balmos's lesson; and
- representative samples of student work from the class.

Using these materials, write a response in which you apply your knowledge of mathematics, mathematics instruction, and mentoring to analyze this case study. Your response should include the following information:

- An analysis of two significant weaknesses in the effectiveness of the lesson on exponential functions. Cite evidence from the case study to support your observations.

- A full description of two instructional strategies or assignments that would be effective for Ms. Balmos to use to address the weaknesses you have identified. Be sure to describe one strategy or assignment for each of the weaknesses you identified.

- An explanation of why each of the strategies or assignments you have described would be effective in improving Ms. Balmos's instruction of exponential functions.

- A full description of two appropriate actions you would take as a mentor teacher to help Ms. Balmos implement the strategies or assignments you have described.

Information from the teacher regarding previous instruction: Students have already studied linear and quadratic functions. This is the first day of a unit on exponential functions.

LESSON PLAN

Objective: Students will investigate the pattern that is the basis of the exponential functions $y = b^x$ and $y = a \bullet b^x$. Students will write equations and apply them to problem solving.

Warm-up

- Have students write two tables of data: one for a linear equation and one for a quadratic equation. Choose a few students to share their tables and graph them on large grid paper in front of the class.

Presentation of Material

- Show tables of various functions and their formulas. Explain the differences between the *y* values.

a)

x	–3	–2	–1	0	1	2
y	–7	–3	1	5	9	13

Linear: $y = 4x + 5$

b)

x	–3	–2	–1	0	1	2
y	6	1	–2	–3	–2	1

Quadratic: $y = x^2 - 3$

c)

x	0	1	2	3	4	5	6
y	1	2	4	8	16	32	64

$y = 2^x$

d)

x	0	1	2	3	4	5	6
y	3	6	12	24	48	96	192

$y = 3 \bullet 2^x$

- State definition of exponential function.
- Demonstrate application problems using exponential formulas. Create tables and find formulas for the following:
  1) A bacteria population doubles every hour. If you start with 10 bacteria, how many will you have in 5 hours? In *x* hours?
  2) Using model of problem #1, how long will it take until there are 10,000 bacteria? Graph with graphing calculator, using trace and table.

LESSON PLAN (continued)

Presentation of Material (continued)	3) A very generous person wins \$1 million in a lottery. She decides to give away her winnings. Every year she looks at worthy causes and picks one to support with donations. In her first year she gives away $\frac{1}{4}$ of her million, and each year after she gives away $\frac{1}{4}$ of what is left. How much of her earnings will she have after 4 years? 8 years? *x* years? 4) Using model of problem #3, how long will it take until she has only one dollar left? Graph, trace, and use table. 5) Roberto, who is 18 years old, invests \$5,000. His money earns interest at 12% per year and is paid into his investment account at the end of each year. By the time he is 65 years old he will have \$1 million (show how to calculate). 6) Compound interest formula: $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$ Use Roberto's \$5,000 in a savings account paying 4% per year, compounded monthly. How much will Roberto have at age 65? 7) Using the information in problem #6, how old would Roberto be by the time his savings account has \$100,000? Assume he takes no money out. Graph, trace, and use table.
Classwork	• Students work in groups of three on "Exponential Functions" assignment.*
Homework	• Finish "Exponential Functions" assignment.
Materials	• Graphing calculator

*A copy of the assignment follows this lesson plan.

CLASSWORK ASSIGNMENT

Exponential Functions

For problems 1–3, find the exponential function.

1)

x	0	1	2	3	4	5
y	1	8	64	512	4096	32768

2)

x	0	1	2	3	4	5
y	5	10	20	40	80	160

3)

x	0	1	2	3	4	5
y	7	21	63	189	567	1701

Solve problems 4–6 using exponential functions.

4) A bacteria population grows so fast that it quadruples each hour. If you start with 1 bacterium, how many will there be in x hours? Construct a table and then write an equation that models this problem. How many bacteria will there be in 8 hours?

5) A ball is dropped from a height of 102 feet. It bounces so that each bounce is $\frac{2}{3}$ the height of the previous bounce. Define 102 feet as bounce zero. How high is bounce 1? Bounce 2? Bounce x?

6) For the ball in problem #5, how high is the tenth bounce?

SELECTED EXCERPTS FROM THE MMT'S OBSERVATION NOTES

- *Students do warm-up individually with ease.*
- *Ms. Balmos chooses three student volunteers to share their work with the class.*
- *Ms. B begins to write the x and y values of Table a on the board. After the fourth value of x, she asks, "What would be the value for y?" Some students call out the correct answer. Ms. B then finishes filling in the table and asks, "How are the y values changing?" This discussion continues, showing why the table shows a linear relationship, and they come up with the equation.*
- *Similarly, Ms. B puts up Table b and a student recognizes that it shows a quadratic relationship and comes up with the equation.*
- *Ms. B puts up Table c and leads discussion that brings up that this table shows neither a linear nor quadratic relationship. Students recognize that the y values are being doubled.*
- *Ms. B agrees and says, "When this happens, this is what the equation looks like," and writes $y = 2^x$ on the board.*
- *Ms. B puts up Table d and asks students to look for the pattern in the y values. Students identify doubling again. Ms. B notes that in this table when the x is zero, the y is 3 instead of 1 like it was in Table c. Ms. B says when the table looks like this, the equation is $y = 3 \cdot 2^x$.*
- *Ms. B begins with one of seven application problems.*
- *With the first application problem, Ms. B asks the students if they could create a table on how the increase in bacteria would occur. Students show understanding--they come up with a table and identify the doubling of the y's as a pattern. Their first attempt at an equation is $y = 2^x$. Despite prompts, the students do not come up with $y = 10 \cdot 2^x$. Ms. B tells them the equation.*

- *On application #2, Ms. B asks how long it will take to reach 10,000. The students have lots of ideas and, with the use of the graphing calculator, Ms. B shows them the answer using "trace" and "table." They find the value of x for which $10 \cdot 2^x = 1{,}000$.*
- *When Ms. B begins the third application problem, she explains what it means to decrease by 25% using various formulas and manipulations to show that $1 - \frac{1}{4}$ simplifies to $\frac{3}{4}$. Students seem to be trying to follow this discussion, but some are lost.*
- *As Ms. B begins the fifth application, she quickly reviews the discussion of percentage increase and decrease. There is notably less student input on this problem. Ms. B continues with the problem.*
- *On the remaining two applications, Ms. B is putting material up on the board with virtually no student input.*
- *Ms. B asks if there are any questions, and there is no response.*
- *Ms. B hands out the assignment as students get into groups of three.*
- *One group is having problems with the differences between $y = b^x$ and $y = a \cdot b^x$. She prompts them to get them to respond.*
- *Ms. B assigns the remaining problems from the assignment sheet for homework.*

SAMPLE STUDENT WORK FROM THE "EXPONENTIAL FUNCTIONS" ASSIGNMENT

The problems below are representative samples of student work from the class.

Student N. N.

1. always times 8, so $y = 8^x$

2. always times 2, so $y = 2^x$

3. always times 3, so $y = 3^x$

4.

x	0	1	2	3
y	1	4	16	64

$y = 4^x$

in 8 hours, $y = 4^8 = 65{,}536$

5.

x	0	1	2
y	102	$\frac{2}{3}(108)$ ≈68	$\frac{2}{3}(68)$ ≈45.3

$$y = \left(\frac{2}{3}\right)^x$$

6. $y = \left(\frac{2}{3}\right)^{10}$

Student B. F.

1. $y = 1 \bullet 8^x$

2. $y = 5 \bullet 2^x$

3. $y = 7 \bullet 3^x$

4.

x	0	1	2	3
y	1	4	8	16

$y = 1 \bullet 4^x$

$y = 4^8 = 65{,}536$ bacteria in 8 hours

5.

x	0	1	2
y	102 ft	68 ft	$\frac{136}{3}$ ft

$y = 102\left(\frac{2}{3}\right)^x$

$y = 68^x$

6. $y = 68^{10}$

$= 2.114 \times 10^{18}$ ft

SECTION VI

PREPARATION RESOURCES

The resources listed below may help you prepare for the TExMaT test in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

Journals

American Mathematical Monthly, Mathematical Association of America.

Journal for Research in Mathematics Education, National Council of Teachers of Mathematics.

Mathematics Magazine, Mathematical Association of America.

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Online Resources

Mathematics TEKS Toolkit, http://www.tenet.edu/teks/math

National Council of Teachers of Mathematics, http://www.nctm.org

Texas Education Agency—Math Initiative, http://www.tea.state.tx.us/math/index.html

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