



TE_xMaT | Texas Examinations for Master Teachers

Preparation Manual



o88 Master Mathematics Teacher 4–8

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PREFACE

The Texas Examinations for Master Teachers (TExMaT) Program has its origins in legislation passed in 1999 (House Bill 2307) that required the creation of the Master Reading Teacher (MRT) Certificate, the development of standards for the certificate, and the development of a Master Reading Teacher examination. In 2001, the Texas legislature passed legislation creating two additional categories of Master Teacher Certificates, the Master Mathematics Teacher (three certificates: Early Childhood–Grade 4, Grades 4–8, and Grades 8–12) and Master Technology Teacher (Early Childhood–Grade 12).

The Master Mathematics Teacher Certificate was created by the 77th Texas Legislature "to ensure that there are teachers with special training to work with other teachers and with students in order to improve student mathematics performance. . . ." A Master Mathematics Teacher will be an individual who holds a Master Mathematics Teacher Certificate and whose primary duties are to teach mathematics and to serve as a mathematics teacher mentor to other teachers.

A Master Mathematics Teacher Certificate may be obtained by individuals who:

- hold a teaching certificate,
- have at least three years of teaching experience,
- complete an SBEC-approved Master Mathematics Teacher preparation program, AND
- pass the TExMaT Master Mathematics Teacher EC–4, 4–8, or 8–12 certification examination.

The development of the educator standards for the Master Mathematics Teacher Certificates was completed in November 2001. The first SBEC-approved Master Mathematics Teacher preparation programs became available during the summer of 2002. The TExMaT Master Mathematics Teacher certification examinations will be administered for the first time in the summer of 2003.

This manual is designed to help examinees prepare for the new Master Mathematics Teacher 4–8 test. Its purpose is to familiarize examinees with the competencies to be tested, test item formats, and pertinent study resources. Educator preparation program staff may also find this information useful as they help examinees prepare for careers as Texas Master Teachers.

More information about the new TExMaT tests and the educator standards can be found at <http://www.sbectx.us>.

KEY FEATURES OF THE MANUAL

List of competencies that will be tested

Strategies for answering test questions

Sample test items and answer key

If you have questions after reading this preparation manual, please contact the State Board for Educator Certification, Office of Accountability at 1-512-238-3200.

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SECTION I

THE NEW TExMAT TESTS

Successful performance on the TExMaT examination is required for the issuance of a Texas Master Teacher certificate. Each TExMaT test is a criterion-referenced examination designed to measure the knowledge and skills delineated in the corresponding TExMaT test framework. Each test framework is based on standards that were developed by Texas educators and other education stakeholders.

Each TExMaT test is designed to measure the requisite knowledge and skills that an initially certified Texas Master Teacher in this field must possess. This test includes multiple-choice items (questions) as well as a case study assignment for which candidates will construct a written response.

Development of the New TExMaT Tests

Committees of Texas educators and interested citizens guide the development of the new TExMaT tests by participating in each stage of the test development process. These working committees are comprised of Texas educators from public and charter schools, faculty from educator preparation programs, education service center staff, representatives from professional educator organizations, content experts, and members of the business community. The committees are balanced in terms of position, affiliation, years of experience, ethnicity, gender, and geographical location. The committee membership is rotated during the development process so that numerous Texas stakeholders may be actively involved. The steps in the process to develop the TExMaT tests are described below.

1. **Develop Standards.** Committees are convened to recommend what an initially certified Master Teacher in this field should know and be able to do. To ensure vertical alignment of standards across the range of instructional levels, individuals with expertise in early childhood, elementary, middle, or high school education meet jointly to articulate the critical knowledge and skills for a particular content area. Participants begin their dialogue using a "clean slate" approach with the Texas Essential Knowledge and Skills (TEKS) as the focal point. Draft standards are written to incorporate the TEKS and to expand upon that content to ensure that an initially certified Master Teacher in this field possesses the appropriate level of both knowledge and skills to instruct successfully.
2. **Review Standards.** Committees review and revise the draft standards. The revised draft standards are then placed on the SBEC Web site for public review and comment. These comments are used to prepare a final draft of the standards that will be presented to the SBEC Board for discussion, the State Board of Education (SBOE) for review and comment, and the SBEC Board for approval.
3. **Develop Test Frameworks.** Committees review and revise draft test frameworks that are based on the standards. These frameworks outline the specific competencies to be measured on the new TExMaT tests. The TExMaT competencies represent the critical components of the standards that can be measured with either a paper-and-pencil-based or a computer-based examination, as appropriate. Draft frameworks are not finalized until after the standards are approved and the job analysis/content validation survey (see #4) is complete.

4. **Conduct Job Analysis/Content Validation Surveys.** A representative sample of Texas educators who practice in or prepare individuals for each of the fields for which a Master Teacher certificate has been proposed are surveyed to determine the relative job importance of each competency outlined in the test framework for that content area. Frameworks are revised as needed following an analysis of the survey responses.
5. **Develop and Review New Test Items.** The test contractor develops draft items (multiple-choice and case study assignments) that are designed to measure the competencies described in the test framework. Committees review the newly developed test items that have been written to reflect the competencies in the new test frameworks and may accept, revise, or reject test items. Committee members scrutinize the draft items for appropriateness of content and difficulty; clarity; match to the competencies; and potential ethnic, gender, and regional bias.
6. **Conduct Pilot Test of New Test Items.** All of the newly developed test items that have been deemed acceptable by the item review committees are then administered to an appropriate sample of candidates for certification.
7. **Review Pilot Test Data.** Pilot test results are reviewed to ensure that the test items are valid, reliable, and free from bias.
8. **Administer New TExMaT Tests.** New TExMaT tests are constructed to reflect the competencies, and the tests are administered to candidates for certification.
9. **Set Passing Standard.** A Standard Setting Committee convenes to review performance data from the initial administration of each new TExMaT test and to recommend a final passing standard for that test. SBEC considers this recommendation as it establishes a passing score on the test.

Taking the TExMaT Master Mathematics Teacher Test and Receiving Scores

Please refer to the current TExMaT registration bulletin for information on test dates, sites, fees, registration procedures, and policies.

You will be mailed a score report approximately four weeks after each test you take. The report will indicate whether you have passed the test and will include:

- a total test *scaled* score. Scaled scores are reported to allow for the comparison of scores on the same content-area test taken on different test administration dates. The total scaled score is not the percentage of items answered correctly and is not determined by averaging the number of questions answered correctly in each domain.
 - For all TExMaT tests, the score scale is 100–300 with a scaled score of 240 as the minimum passing score. This score represents the minimum level of competency required to be a Master Teacher in this field in Texas public schools.
- a holistic score for your response to the case study assignment.
- your performance in the major content domains of the test and in the specific content competencies of the test.
 - This information may be useful in identifying strengths and weaknesses in your content preparation and can be used for further study or for preparing to retake the test.
- information to help you understand the score scale and interpret your results.

You will not receive a score report if you are absent or choose to cancel your score.

Additionally, unofficial score report information will be posted on the Internet on the score report mailing date of each test administration. Information about receiving unofficial scores via the Internet and other score report topics may be found on the SBEC Web site at www.sbec.state.tx.us.

Educator Standards

Complete, approved educator standards are posted on the SBEC Web site at www.sbec.state.tx.us.

SECTION II

USING THE TEST FRAMEWORK

The Texas Examinations for Master Teachers (TExMaT) test measures the content and professional knowledge required of an initially certified Master Teacher in this field. This manual is designed to guide your preparation by helping you become familiar with the material to be covered on the test.

When preparing for this test, you should focus on the competencies and descriptive statements, which delineate the content that is eligible for testing. A portion of the content is represented in the sample items that are included in this manual. These test questions represent only a *sample* of items. Thus, your test preparation should focus on the complete content eligible for testing, as specified in the competencies and descriptive statements.

Organization of the TExMaT Test Framework

The test framework is based on the educator standards for this field.

The content covered by this test is organized into broad areas of content called domains. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of competencies. Each competency is composed of two major parts:

1. the *competency statement*, which broadly defines what an initially certified Master Teacher in this field should know and be able to do, and
2. the *descriptive statements*, which describe in greater detail the knowledge and skills eligible for testing.

The educator standards being assessed within each domain are listed for reference at the beginning of the test framework. These are then followed by a complete set of the framework's competencies and descriptive statements.

An example of a competency and its accompanying descriptive statements is provided on the next page.

Sample Competency and Descriptive Statements

Master Mathematics Teacher 4–8

Competency:

The Master Mathematics Teacher 4–8 understands the structure of number systems, the development of a sense of quantity, and the relationship between quantity and symbolic representations.

Descriptive Statements:

The Master Mathematics Teacher:

- Analyzes the structure of numeration systems and the roles of place value and zero in the base ten system.
- Demonstrates an understanding of a variety of models for representing numbers (e.g., fraction strips, diagrams, patterns, shaded regions, number lines).
- Demonstrates an understanding of equivalency among different representations of rational numbers.
- Selects appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- Understands the characteristics of the sets of whole numbers, integers, rational numbers, real numbers, and complex numbers (e.g., order, commutativity, closure, identity elements, inverse elements, density).
- Demonstrates an understanding of how some situations that have no solutions in one number system (e.g., whole numbers, integers, rational numbers) have solutions in another number system (e.g., real numbers, complex numbers).

Studying for the TExMaT Test

The following steps may be helpful in preparing for the TExMaT test.

1. Identify the information the test will cover by reading through the test competencies (see the following pages in this section). *Within each domain* of this TExMaT test, each competency will receive approximately equal coverage.
2. Read each competency with its descriptive statements in order to get a more specific idea of the knowledge you will be required to demonstrate on the test. You may wish to use this review of the competencies to set priorities for your study time.
3. Review the "Preparation Resources" section of this manual for possible resources to consult. Also, compile key materials from your preparation coursework that are aligned with the competencies.
4. Study this manual for approaches to taking the test.
5. When using resources, concentrate on the key ideas and important concepts that are discussed in the competencies and descriptive statements.

NOTE: This preparation manual is the only TExMaT test study material endorsed by SBEC for this field. Other preparation materials may not accurately reflect the content of the test or the policies and procedures of the TExMaT Program.

TEST FRAMEWORK FOR MASTER MATHEMATICS TEACHER 4–8

Domain I **Number Concepts: Content, Instruction, and Assessment** (approximately 18.5% of the test)

Standards Assessed:

Standard I: Number Concepts: The Master Mathematics Teacher understands and applies knowledge of numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and the vertical alignment of number concepts to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

Domain II **Patterns and Algebra: Content, Instruction, and Assessment** (approximately 26% of the test)

Standards Assessed:

Standard II: Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

**Domain III Geometry and Measurement: Content, Instruction, and Assessment
(approximately 22% of the test)**

Standards Assessed:

Standard III: Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

**Domain IV Probability and Statistics: Content, Instruction, and Assessment
(approximately 18.5% of the test)**

Standards Assessed:

Standard IV: Probability and Statistics: The Master Mathematics Teacher understands probability and statistics, their applications, and the vertical alignment of probability and statistics to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard VI: Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII: Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII: Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

**Domain V Mathematical Processes, Perspectives, Mentoring, and Leadership
(approximately 15% of the test)**

Standards Assessed:

Standard V: Mathematical Processes: The Master Mathematics Teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.

Standard IX: Mentoring and Leadership: The Master Mathematics Teacher facilitates appropriate standards-based mathematics instruction by communicating and collaborating with educational stake-holders; mentoring, coaching, exhibiting leadership, and consulting with colleagues; providing professional development opportunities for faculty; and making instructional decisions based on data and supported by evidence from research.

Standard X: Mathematical Perspectives: The Master Mathematics Teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics, and the evolving nature of mathematics and mathematical knowledge.

DOMAIN I—NUMBER CONCEPTS: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 001

The Master Mathematics Teacher 4–8 understands the structure of number systems, the development of a sense of quantity, and the relationship between quantity and symbolic representations.

The Master Mathematics Teacher:

- Analyzes the structure of numeration systems and the roles of place value and zero in the base ten system.
- Demonstrates an understanding of a variety of models for representing numbers (e.g., fraction strips, diagrams, patterns, shaded regions, number lines).
- Demonstrates an understanding of equivalency among different representations of rational numbers.
- Selects appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- Understands the characteristics of the sets of whole numbers, integers, rational numbers, real numbers, and complex numbers (e.g., order, commutativity, closure, identity elements, inverse elements, density).
- Demonstrates an understanding of how some situations that have no solutions in one number system (e.g., whole numbers, integers, rational numbers) have solutions in another number system (e.g., real numbers, complex numbers).

Competency 002

The Master Mathematics Teacher 4–8 understands number operations and computational algorithms.

The Master Mathematics Teacher:

- Works proficiently with real and complex numbers and their operations.
- Analyzes and describes relationships among number properties, operations, and algorithms for the four basic operations involving integers and rational, real, and complex numbers.
- Uses a variety of concrete and visual representations, and technology where appropriate, to investigate the connections between operations and algorithms.
- Justifies procedures used in algorithms for the four basic operations with integers, rational numbers, and real numbers, and analyzes error patterns that may occur in their application.
- Relates operations and algorithms involving numbers to algebraic procedures.
- Understands and applies the use of rational numbers as exponents.

Competency 003

The Master Mathematics Teacher 4–8 understands ideas of number theory and uses numbers to model and solve problems within and outside of mathematics, using technology as appropriate.

The Master Mathematics Teacher:

- Demonstrates an understanding of ideas from number theory (e.g., prime factorization, greatest common divisor, division algorithm) as they apply to whole numbers, integers, and rational numbers, and uses these ideas in problem situations.
- Uses integers, rational numbers, and real numbers to describe and quantify phenomena such as money, length, area, volume, and density.
- Applies knowledge of place value and other number properties to develop techniques of mental mathematics and computational estimation.
- Applies knowledge of counting techniques, such as permutations and combinations, to quantify situations and solve problems.
- Applies properties of the real numbers to solve a variety of theoretical and applied problems.

Competency 004

The Master Mathematics Teacher 4–8 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop number concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use number concepts.
- Recognizes and uses the vertical alignment of number concepts across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of number concepts.
- Uses evidence of students' current understanding of number concepts to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within number concepts.
- Designs challenging and engaging problem-solving tasks that develop number-concepts content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates number concepts within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of number concepts.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 005

The Master Mathematics Teacher 4–8 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of number concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve number concepts, skills, and procedures.
- Knows how to teach number concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of number concepts.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning number concepts.
- Evaluates and modifies instruction to improve learning of number concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN II—PATTERNS AND ALGEBRA: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 006

The Master Mathematics Teacher 4–8 understands and uses mathematical reasoning to identify, extend, and analyze patterns and understands the relationships among variables, expressions, equations, inequalities, relations, and functions, using technology as appropriate.

The Master Mathematics Teacher:

- Uses inductive reasoning to identify, extend, generalize, and create patterns using concrete models, geometric figures, numbers, and algebraic expressions.
- Formulates implicit and explicit rules to describe and construct sequences verbally, numerically, graphically, and symbolically.
- Makes, tests, validates, and uses conjectures about patterns and relationships in data presented in tables, sequences, or graphs.
- Uses methods of recursion and iteration to model and solve problems.
- Analyzes the properties of sequences and series and uses them to solve problems involving finite and infinite processes, including problems related to simple and compound interest rates, and average and instantaneous rates of change.
- Gives appropriate justification of the manipulation of algebraic expressions, equations, and inequalities.
- Analyzes attributes of functions and relations (e.g., domain, range, one-to-one functions, inverse functions) and their graphs.
- Illustrates the concept of a function using concrete models, tables, graphs, and verbal and symbolic representations.
- Uses transformations to illustrate properties of functions and relations and to solve problems.

Competency 007

The Master Mathematics Teacher 4–8 understands and uses linear functions to model and solve problems using a variety of methods, including technology.

The Master Mathematics Teacher:

- Demonstrates an understanding of the concept of linear function using concrete models, tables, graphs, and verbal and symbolic representations.
- Demonstrates an understanding of the connections among linear functions, proportions, and direct variation.
- Determines a linear function that models a set of data.
- Analyzes the relationships among a linear function, its average rate of change, and its graph.
- Uses linear functions, inequalities, and systems to model real-world situations.
- Uses a variety of representations and methods (e.g., numerical methods, tables, graphs, algebraic techniques) to solve systems of linear equations and inequalities.
- Uses transformations to illustrate properties of linear functions and to solve problems.
- Describes and predicts the effects of transformations on the graphs of linear functions.
- Demonstrates an understanding of the characteristics of linear models and the advantages and disadvantages of using a linear model in a given situation.

Competency 008

The Master Mathematics Teacher 4–8 understands and uses quadratic functions and relations to model and solve problems using a variety of methods, including technology.

The Master Mathematics Teacher:

- Uses a variety of methods to investigate the roots (real and complex), vertex, and symmetry of a quadratic function or relation.
- Demonstrates an understanding of the connections among geometric, graphic, numeric, and symbolic representations of quadratic functions.
- Uses transformations to illustrate properties of quadratic functions and relations and to solve problems.
- Understands the effects of transformations on the graph of a quadratic function.
- Uses quadratic functions to analyze, model, and solve problems.
- Uses a variety of representations and methods (e.g., numerical methods, tables, graphs, algebraic techniques) to solve systems of equations and inequalities involving quadratics.

Competency 009

The Master Mathematics Teacher 4–8 understands and uses nonlinear functions and relations, including polynomial, absolute value, trigonometric, rational, radical, exponential, logarithmic, and piecewise-defined functions, to model and solve problems using a variety of methods, including technology.

The Master Mathematics Teacher:

- Demonstrates an understanding of the connections among geometric, graphic, numeric, and symbolic representations of nonlinear functions.
- Uses transformations to illustrate properties of nonlinear functions and relations and to solve problems.
- Understands the effects of transformations on the graph of a nonlinear function.
- Analyzes data and represents and solves problems involving exponential growth and decay.
- Demonstrates an understanding of the connections among proportions, inverse variation, and rational functions.
- Applies properties and graphs of nonlinear functions to analyze, model, and solve problems.

Competency 010

The Master Mathematics Teacher 4–8 understands and uses the conceptual foundations of calculus related to topics in middle school mathematics.

The Master Mathematics Teacher:

- Relates topics in middle school mathematics to the concept of limit in sequences and series.
- Relates the concept of average rate of change to the slope of the secant line, and relates instantaneous rate of change to the slope of the tangent line.
- Relates topics in middle school mathematics to the area under a curve.
- Demonstrates an understanding of the use of calculus concepts to answer questions about rates of change, areas, volumes, and properties of functions and their graphs.

Competency 011

The Master Mathematics Teacher 4–8 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop patterns and algebra concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use patterns and algebra.
- Recognizes and uses the vertical alignment of patterns and algebra across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that students develop accuracy and fluency of patterns and algebra.
- Uses evidence of students' current understanding of patterns and algebra to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within patterns and algebra.
- Designs challenging and engaging problem-solving tasks that develop patterns and algebra content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates patterns and algebra concepts within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of patterns and algebra.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 012

The Master Mathematics Teacher 4–8 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of patterns and algebra concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve patterns and algebra concepts, skills, and procedures.
- Knows how to teach patterns and algebra concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of patterns and algebra concepts.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning patterns and algebra concepts.
- Evaluates and modifies instruction to improve learning of patterns and algebra concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN III—GEOMETRY AND MEASUREMENT: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 013

The Master Mathematics Teacher 4–8 understands measurement as a process.

The Master Mathematics Teacher:

- Selects and uses appropriate units of measurement to quantify, compare, and communicate information (e.g., temperature, money, mass, weight, area, capacity, density, percentage, speed, acceleration).
- Justifies and uses conversions within measurement systems.
- Applies dimensional analysis in a variety of situations to derive units and formulas and to find and evaluate solutions to problems.
- Describes the precision of measurement and the effects of error on measurement.
- Applies the Pythagorean theorem, proportional reasoning, and right-triangle trigonometry to solve measurement problems.

Competency 014

The Master Mathematics Teacher 4–8 understands the geometric relationships and axiomatic structure of Euclidean geometry.

The Master Mathematics Teacher:

- Recognizes axiomatic systems and their components, such as undefined terms, defined terms, theorems, examples, and counterexamples.
- Understands concepts and properties of points, lines, planes, angles, lengths, and distances.
- Analyzes and applies the properties of parallel and perpendicular lines.
- Uses the properties of similar and congruent triangles to explore geometric relationships and prove theorems.
- Describes and justifies geometric constructions made using a compass and straightedge and other appropriate technologies.
- Applies knowledge of the axiomatic structure of Euclidean geometry to justify and prove theorems.

Competency 015

The Master Mathematics Teacher 4–8 analyzes the properties of two- and three-dimensional figures.

The Master Mathematics Teacher:

- Understands and uses the development of formulas to find lengths, perimeters, areas, and volumes of basic geometric figures.
- Applies relationships among similar figures, scale, and proportion and analyzes how changes in scale affect area and volume measurements.
- Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two- and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms, and spheres.
- Analyzes the relationships among three-dimensional figures and related two-dimensional representations (e.g., projections, cross sections, nets) and uses these representations to solve problems.
- Investigates and explores geometric concepts and properties using technology.

Competency 016

The Master Mathematics Teacher 4–8 understands transformational and vector geometry, and relates algebra to geometry and trigonometry using the Cartesian coordinate system.

The Master Mathematics Teacher:

- Describes and justifies geometric constructions made using a reflection device and other appropriate technologies.
- Uses translations, reflections, glide-reflections, and rotations to demonstrate congruence and to explore the symmetries of figures.
- Uses dilations (expansions and contractions) to illustrate similar figures and proportionality.
- Uses symmetry to describe tessellations and shows how they can be used to illustrate geometric concepts, properties, and relationships.
- Applies concepts and properties of slope, midpoint, parallelism, and distance in the coordinate plane to explore properties of geometric figures and solve problems.
- Applies transformations in the coordinate plane.
- Uses the unit circle in the coordinate plane to explore properties of trigonometric functions using both degrees and radians.
- Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.
- Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

Competency 017

The Master Mathematics Teacher 4–8 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop geometry and measurement concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use geometry and measurement.
- Recognizes and uses the vertical alignment of geometry and measurement across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of geometry and measurement.
- Uses evidence of students' current understanding of geometry and measurement to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within geometry and measurement.
- Designs challenging and engaging problem-solving tasks that develop geometry and measurement content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates geometry and measurement within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of geometry and measurement.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 018

The Master Mathematics Teacher 4–8 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of geometry and measurement concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve geometry and measurement concepts, skills, and procedures.
- Knows how to teach geometry and measurement concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of geometry and measurement.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning geometry and measurement.
- Evaluates and modifies instruction to improve learning of geometry and measurement concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN IV—PROBABILITY AND STATISTICS: CONTENT, INSTRUCTION, AND ASSESSMENT

Competency 019

The Master Mathematics Teacher 4–8 understands how to use graphical and numerical techniques to explore data, characterize patterns, and describe departures from patterns.

The Master Mathematics Teacher:

- Organizes and displays data in a variety of formats (e.g., tables, frequency distributions, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).
- Applies concepts of center, spread, shape, and skewness to describe a data distribution.
- Supports arguments, makes predictions, and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.
- Demonstrates an understanding of measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, interquartile range, variance, standard deviation).
- Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers, and measures of central tendency and dispersion.
- Calculates and interprets percentiles and quartiles.

Competency 020

The Master Mathematics Teacher 4–8 understands and applies the concepts of probability.

The Master Mathematics Teacher:

- Explores concepts of probability through data collection, experiments, and simulations.
- Calculates the probabilities of simple and compound events using concepts of probability (e.g., addition rule, multiplication rule, conditional probability, independence).
- Chooses a simulation or uses a probability distribution to represent a situation and solve related problems.
- Determines probabilities by constructing sample spaces.
- Solves a variety of probability problems using combinations, permutations, and geometric probability (i.e., probability as the ratio of two areas).
- Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, normal).

Competency 021

The Master Mathematics Teacher 4–8 understands the relationship among probability theory, sampling, and statistical inference, and understands how statistical inference is used in making and evaluating predictions.

The Master Mathematics Teacher:

- Applies knowledge of designing, conducting, analyzing, and interpreting statistical experiments to investigate real-world problems.
- Analyzes and interprets statistical information from the media (e.g., polls, surveys) and recognizes valid and misleading uses of statistics.
- Demonstrates an understanding of random samples, sample statistics, and the relationship between sample size and confidence intervals.
- Applies knowledge of the use of probability to make observations about and draw conclusions from one-variable data, and to describe the level of confidence in the conclusion, including the appropriate use of technology.
- Makes inferences about a population using binomial, geometric, and normal distributions.
- Demonstrates an understanding of the use of techniques such as scatterplots, regression lines, correlation coefficients, and residual analysis to explore bivariate data and to make and evaluate predictions, including the appropriate use of technology.

Competency 022

The Master Mathematics Teacher 4–8 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop probability and statistics concepts, skills, and procedures.

The Master Mathematics Teacher:

- Evaluates and applies established research evidence on how all students, including students who are at-risk, learn and use probability and statistics.
- Recognizes and uses the vertical alignment of probability and statistics across grade levels to plan instruction based on state standards.
- Sequences instruction, practice, and applications based on students' instructional needs so that all students develop accuracy and fluency of probability and statistics.
- Uses evidence of students' current understanding of probability and statistics to select strategies to help students move from informal to formal knowledge.
- Structures problem-solving activities so students can recognize patterns and relationships within probability and statistics.
- Designs challenging and engaging problem-solving tasks that develop probability and statistics content knowledge as well as students' critical and analytical reasoning capacities.
- Integrates probability and statistics within and outside of mathematics.
- Selects appropriate materials, instructional strategies, and technology to meet the instructional needs of all students.
- Uses strategies to help students understand that results obtained using technology may be misleading and/or misinterpreted.
- Recognizes common errors and misconceptions and determines appropriate correction procedures.
- Develops assessments based on state and national standards to evaluate students' knowledge of probability and statistics.
- Evaluates an assessment for validity with respect to the measured objectives.
- Analyzes and uses assessment results from various diagnostic instruments to plan, inform, and adjust instruction.
- Recognizes how to provide equity for all students in mathematics instruction through reflection on one's own attitudes, expectations, and teaching practices.

Competency 023

The Master Mathematics Teacher 4–8 implements a variety of instruction and assessment techniques to guide, evaluate, and improve students' learning of probability and statistics concepts, skills, and procedures.

The Master Mathematics Teacher:

- Creates a positive learning environment that provides all students with opportunities to develop and improve probability and statistics concepts, skills, and procedures.
- Knows how to teach probability and statistics concepts, skills, procedures, and problem-solving strategies using instructional approaches supported by established research.
- Knows how to maximize student/teacher and student/student interaction and analyzes students' abilities to correctly apply new content.
- Uses multiple representations, tools, and a variety of tasks to promote students' understanding of probability and statistics.
- Introduces content by carefully defining new terms using vocabulary that the student already knows.
- Uses a variety of questioning strategies to identify, support, monitor, and challenge students' mathematical thinking.
- Demonstrates classroom management skills, including applying strategies that use instructional time effectively.
- Administers a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns associated with learning probability and statistics.
- Evaluates and modifies instruction to improve learning of probability and statistics concepts, skills, and procedures for all students based on the results of formal and informal assessments.

DOMAIN V—MATHEMATICAL PROCESSES, PERSPECTIVES, MENTORING, AND LEADERSHIP

Competency 024

The Master Mathematics Teacher 4–8 understands and uses mathematical processes to reason mathematically and solve problems.

The Master Mathematics Teacher:

- Demonstrates an understanding of the use of logical reasoning to evaluate mathematical conjectures and justifications and to provide convincing arguments or proofs for mathematical theorems.
- Applies correct mathematical reasoning to derive valid conclusions from a set of premises, and recognizes examples of fallacious reasoning.
- Demonstrates an understanding of the use of inductive reasoning to make conjectures and deductive methods to evaluate the validity of conjectures.
- Applies knowledge of the use of formal and informal reasoning to explore, investigate, and justify mathematical ideas.
- Recognizes that a mathematical problem can be solved in a variety of ways and selects an appropriate strategy for a given problem.
- Evaluates the reasonableness of a solution to a given problem.
- Demonstrates an understanding of estimation and evaluates its appropriate uses.
- Uses physical and numerical models to represent a given problem or mathematical procedure.
- Recognizes that assumptions are made when solving problems; then identifies and evaluates those assumptions.
- Investigates and explores problems that have multiple solutions.
- Applies content knowledge to develop a mathematical model of a real-world situation; then analyzes and evaluates how well the model represents the situation.
- Develops and uses simulations as a tool to model and solve problems.

Competency 025

The Master Mathematics Teacher 4–8 understands mathematical connections, the structure of mathematics, the historical development of mathematics, and how to communicate mathematical ideas and concepts.

The Master Mathematics Teacher:

- Recognizes and uses multiple representations of a mathematical concept.
- Uses mathematics to model and solve problems in other disciplines.
- Uses the structure of mathematical systems and their properties (e.g., mappings, inverse operations) to make connections among mathematical concepts.
- Recognizes the impacts of technological advances on mathematics (e.g., numerical versus analytical solutions) and of mathematics on technology (binary arithmetic).
- Emphasizes the role of mathematics in various careers and professions (e.g., economics, engineering) and how technology (e.g., spreadsheets, statistical software) affects the use of mathematics in various careers.
- Knows and uses the history and evolution of mathematical concepts, procedures, and ideas (e.g., the development of non-Euclidean geometry).
- Recognizes the contributions that different cultures have made to the field of mathematics.
- Uses current and professional resources to plan and develop activities that provide cultural, historical, and technological instruction for the classroom and that connect society and mathematics.
- Expresses mathematical statements using developmentally appropriate language, standard English, mathematical language, and symbolic mathematics.
- Communicates mathematical ideas using a wide range of technological tools and a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).
- Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams, and animations to communicate mathematical information.
- Uses the language of mathematics as a precise means of expressing mathematical ideas.

Competency 026

The Master Mathematics Teacher 4–8 knows how to communicate and collaborate with educational stakeholders to facilitate implementation of appropriate, standards-based mathematics instruction.

The Master Mathematics Teacher:

- Knows the dual role of the Master Mathematics Teacher as teacher and mentor in the school community.
- Knows leadership, communication, and facilitation skills and strategies.
- Knows and applies principles, guidelines, and professional ethical standards regarding collegial and professional collaborations, including issues related to confidentiality.
- Understands the importance of collaborating with administrators, colleagues, parents/guardians, and other members of the school community to establish and implement the roles of the Master Mathematics Teacher and ensure effective ongoing communication.
- Knows strategies for communicating effectively with stakeholders, including other teachers, about using programs and instructional techniques that are based on established research that supports their effectiveness with a range of students, including students who are at-risk.
- Knows strategies for building trust and a spirit of collaboration with other members of the school community to effect positive change in the school mathematics program and mathematics instruction.
- Knows how to use leadership skills to ensure the effectiveness and ongoing improvement of the school mathematics program, encourage support for the program, and engage others in improving the program.
- Knows strategies for collaborating with members of the school community to evaluate, negotiate, and establish priorities regarding the mathematics program and to facilitate mentoring, professional development, and parent/guardian training.
- Knows strategies for conferring with students, colleagues, administrators, and parents/guardians to discuss mathematics-related issues.
- Knows strategies for collaborating with teachers, administrators, and others to identify professional development needs, generate support for professional development programs, and ensure provision of effective professional development opportunities.

Competency 027

The Master Mathematics Teacher 4–8 knows how to provide professional development through mentoring, coaching, and consultation with colleagues to facilitate implementation of appropriate, standards-based mathematics instruction, and makes instructional decisions supported by established research.

The Master Mathematics Teacher:

- Knows and applies skills and strategies for mentoring, coaching, and consultation in the development, implementation, and evaluation of an effective mathematics program.
- Knows learning processes and procedures for facilitating adult learning.
- Knows strategies for facilitating positive change in instructional practices through professional development, mentoring, coaching, and consultation.
- Knows models and features of effective professional development programs that promote sustained applications in classroom practice (e.g., modeling, coaching, follow-up).
- Knows differences between consultation and supervision.
- Knows how to use mentoring, coaching, and consultation to facilitate team building for promoting student development in mathematics.
- Knows how to select and use strategies for collaborating with colleagues to identify needs related to mathematics instruction.
- Knows strategies for collaborating effectively with colleagues with varying levels of skill and experience and/or diverse philosophical approaches to mathematics instruction to develop, implement, and monitor mathematics programs.
- Knows how to select and use strategies to maximize effectiveness as a Master Mathematics Teacher, such as applying principles of time management and engaging in continuous self-assessment.
- Knows sources for locating information about established research on mathematics learning and understands methods and criteria for reviewing research on mathematics learning.
- Knows how to critically examine established research on mathematics learning, analyzes its usefulness for addressing instructional needs, and applies appropriate procedures for translating research on mathematics learning into practice.

SECTION III

APPROACHES TO ANSWERING MULTIPLE-CHOICE ITEMS

The purpose of this section is to describe multiple-choice item formats that you will see on the TExMaT Master Mathematics Teacher (MMT) test and to suggest possible ways to approach thinking about and answering the multiple-choice items. However, these approaches are not intended to replace familiar test-taking strategies with which you are already comfortable and that work for you.

The Master Mathematics Teacher 4–8 test is designed to include 80 scorable multiple-choice items and approximately 10 nonscorable items. Your final scaled score will be based only on scorable items. The nonscorable multiple-choice items are pilot tested by including them in the test in order to collect information about how these questions will perform under actual testing conditions. Nonscorable test items are not considered in calculating your score, and they are not identified on the test.

All multiple-choice questions on this test are designed to assess your knowledge of the content described in the test framework. The multiple-choice questions assess your ability to recall factual information **and** to think critically about the information, analyze it, consider it carefully, compare it with other knowledge you have, or make a judgment about it.

When you are ready to answer a multiple-choice question, you must choose one of four *answer choices* labeled A, B, C, and D. Then you must mark your choice on a separate answer sheet.

In addition to the multiple-choice questions, the MMT test will include one case study assignment. Please see Section V: Case Study Assignment.

Calculators. If you want to use a calculator, you must bring your own calculator to the test administration. However, only the brands and models listed in the TExMaT registration bulletin may be used at the test. All calculators on the approved list are graphing calculators. Graphing calculators perform all the operations of typical scientific calculators. **Test administration staff will clear the memory of your calculator both before and after the test.**

NOTE: Some test questions for Master Mathematics Teacher 4–8 are designed to be solved with a graphing calculator. It is therefore strongly recommended that you bring a graphing calculator with you to the test site. Sharing of calculators will not be permitted.

The approved calculator brands and models are subject to change. If there is a change, examinees will be notified.

Calculators may be used for both the multiple-choice and case study sections of the test.

Definitions and Formulas. A set of definitions and formulas will be provided in your test booklet. A copy of those definitions and formulas is also provided in Section IV of this preparation manual.

Multiple-Choice Item Formats

You may see the following two types of multiple-choice questions on the test.

- Single items
- Items with stimulus material

You may have two or more items related to a single stimulus. This group of items is called a cluster. Following the last item of a clustered item set containing two or more items, you will see the graphic illustrated below.



This graphic is used to separate these clustered items related to specific stimulus material from other items that follow.

On the following pages, you will find descriptions of these commonly used item formats, along with suggested approaches for answering each type of item. In the actual testing situation, you may mark the test items and/or write in the margins of your test booklet, **but your final response must be indicated on the answer sheet provided.**

SINGLE ITEMS

In the single item format, a problem is presented as a direct question or an incomplete statement, and four answer choices appear below the question. The following question is an example of this type. It tests knowledge of Master Mathematics Teacher 4–8 competency 004: *The Master Mathematics Teacher 4–8 plans and designs effective instruction and assessment based on knowledge of how all students, including students who are at-risk, learn and develop number concepts, skills, and procedures.*

Which of the following activities would most directly enhance students' understanding of factoring?

- A. Each student draws a 4×4 grid and then counts how many squares of all possible sizes occur within the grid. Answers are then compared.
 - B. Each student draws a 4×4 grid and is asked to find how many different ways it is possible for a person located at point A to move to point B.
 - C. Each student is given 55 cubes with 1 cm edges and builds a staircase. A rule is discovered for predicting how many cubes it will take to build a staircase with 70 steps.
 - D. Each student is given 48 cubes with 1 cm edges and is asked to build a rectangular prism. The dimensions of the resulting prisms are compared.
-

Suggested Approach

Read the question carefully and critically. Think about what it is asking and the situation it is describing. Eliminate any obviously wrong answers, select the correct answer choice, and mark it on your answer sheet.

With guidance from the teacher, the students will discover that rectangular prisms with different dimensions can be constructed. For example, one student might create a string of 48 cubes (a $48 \times 1 \times 1$ prism), while another student could build a tower 8 cubes high, on a 2 by 3 cube rectangular base (a $2 \times 3 \times 8$ prism). Comparison of the different rectangular prisms that can be built will lead to the discovery that only sides of certain integer lengths are possible, i.e., 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48. This activity will directly enhance students' understanding of factoring because the fact that a rectangular prism of specified volume can be built using only certain combinations of integer side lengths is conceptually similar to the fact that an integer can be generated using only certain combinations of factors.

Option A and option B describe activities that would be useful for enhancing visualization, counting, and problem-solving skills. The activity in option C involves the extension of patterns geometrically, numerically, and algebraically. None of these options directly addresses the concept of factoring. Therefore, the correct response is option D.

ITEMS WITH STIMULUS MATERIAL

Some questions are preceded by stimulus material that relates to the item. Some types of stimulus material included on the test are reading passages, graphics, tables, or a combination of these. In such cases, you will generally be given information followed by an event to analyze, a problem to solve, or a decision to make.

One or more items may be related to a single stimulus. You can use several different approaches to answer these types of questions. Some commonly used approaches are listed below.

- Strategy 1** Skim the stimulus material to understand its purpose, its arrangement, and/or its content. Then read the item and refer again to the stimulus material to verify the correct answer.
- Strategy 2** Read the item *before* considering the stimulus material. The content of the item will help you identify the purpose of the stimulus material and locate the information you need to answer the question.
- Strategy 3** Use a combination of both strategies; apply the "read the stimulus first" strategy with shorter, more familiar stimuli and the "read the item first" strategy with longer, more complex, or less familiar stimuli. You can experiment with the sample items in this manual and then use the strategy with which you are most comfortable when you take the actual test.

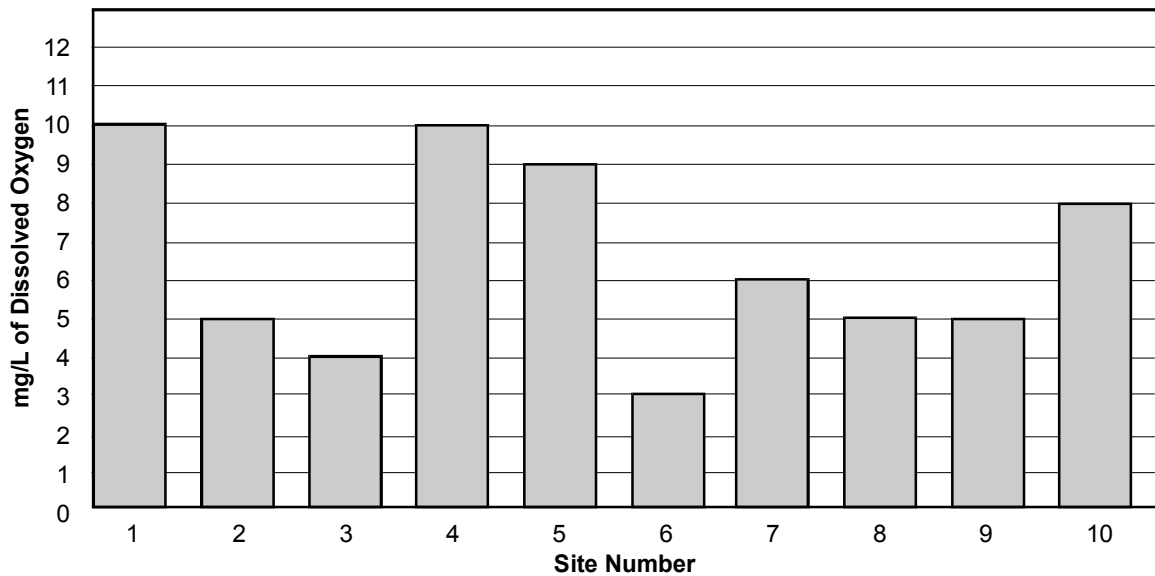
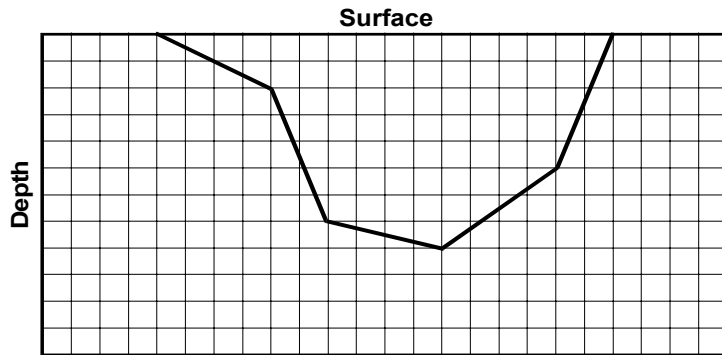
Whether you read the stimulus before or after you read the item, you should read it carefully and critically. You may want to underline its important points to help you answer the item.

As you consider items set in educational contexts, try to use the identified teacher's point of view to answer the items that accompany the stimulus. Be sure to consider the items in terms of only the information provided in the stimulus—not in terms of specific situations or individuals you may have encountered.

Suggested Approach

First read the stimulus (information and data from the study of a stream conducted by students). A sample stimulus is shown below.

Use the diagrams below to answer the three questions that follow.



Students are studying a straight, 15-foot section of stream near their school. Figure 1 shows a typical cross-section of the stream. The boxes of the grid represent one-inch squares. Figure 2 shows the quantities of dissolved oxygen present in water samples from 10 different sites in the stream.

Now you are prepared to address the first of the two questions associated with this stimulus. The first question measures Master Mathematics Teacher 4–8 competency 0015: *The Master Mathematics Teacher 4–8 analyzes the properties of two- and three-dimensional figures.*

A rubber ball dropped in the water moves 3 feet downstream in 12 minutes. What is, in cubic inches per hour, the flow rate of the stream?

- A. 11,520
- B. 13,320
- C. 13,860
- D. 14,400

An estimate of the volume of the stream is needed in order to calculate its flow rate. The first step is to calculate the area of the cross-section of the stream using Figure 1.

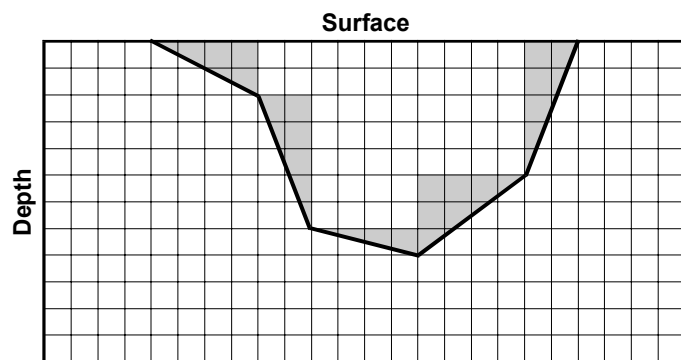


Figure 1

The area of the cross-section of the stream can be calculated as the sum of the unshaded area in the interior of the stream in the figure and the total area of the five shaded triangles. The area of each of these triangles can be found using the formula for the area of a triangle, $\text{area} = \frac{1}{2} \times \text{base} \times \text{height}$. The total area of the shaded triangles is $\frac{1}{2} \times (4 \cdot 2 + 2 \cdot 5 + 4 \cdot 1 + 4 \cdot 3 + 2 \cdot 5) = 22$ square inches and the area of the cross-section of the stream is $52 + 22 = 74$ square inches.

The rate at which the water is moving is 3 feet per 12 minutes. This converts to 15 feet per 60 minutes, which is 15 feet per hour, or $15 \cdot 12 = 180$ inches per hour. If 74 square inches of water are moving at a rate of 180 inches per hour, the flow rate can be calculated as $74 \times 180 = 13,320$ cubic inches per hour.

The correct response is option B. Option A results from calculating the cross-section of the stream as the area of one large triangle $\left(\frac{1}{2} \times 16 \times 8\right)$, option C from calculating the cross-section as the area of a large trapezoid $\left[\frac{1}{2} \times (16 + 6) \times 7\right]$, and option D from calculating the cross-section as the area of a large rectangle (10×8) .

Now you are ready to answer the next question. The second question measures competency 0013: *The Master Mathematics Teacher 4–8 understands measurement as a process.*

What is the approximate surface area in square feet of the section of the stream being studied?

- A. 15
 - B. 19
 - C. 20
 - D. 21
-

Since the section of the stream is straight, its surface area can be represented by a rectangle 15 feet long and 16 inches wide, based on the cross-section in Figure 1. The width can be expressed as $1\frac{1}{3}$ feet, so the area is $\frac{4}{3} \times 15 = 20$ square feet.

Of the four options offered, only option C is correct. Option A rounds the width of the stream to 1 foot before multiplying by the length, 15 feet. Option B represents the stream width, 1 foot and 4 inches, as $\frac{5}{4}$ of a foot, and option D calculates the surface area as the sum of the actual surface area and the cross-section.

Now you are ready to answer the next question. The third question measures competency 0019: *The Master Mathematics Teacher 4–8 understands how to use graphical and numerical techniques to explore data, characterize patterns, and describe departures from patterns.*

What is the difference in milligrams per liter between the mean and the median dissolved oxygen values for the data in Figure 2?

- A. 0
 - B. 0.5
 - C. 1.0
 - D. 1.5
-

Use the data presented in Figure 2 to calculate the mean milligrams of oxygen per liter. Sum the values for the 10 sites and divide by the number of sites (10). Thus the mean value is

$$\frac{10 + 5 + 4 + 10 + 9 + 3 + 6 + 5 + 5 + 8}{10} = 6.5 \text{ milligrams of oxygen per liter.}$$

The median is the middle point of the data distribution, i.e., half the data points are above it, and half below it. Arrange the data values from high to low as follows: 10, 10, 9, 8, 6, 5, 5, 5, 4, 3. Since there are an even number of values, none of the data values represents the true middle point of the data. In this case, take the average of the two middle points, 5 and 6, as the median. Thus the median is 5.5 milligrams of oxygen per liter, and the difference between the mean and median values is $6.5 - 5.5 = 1.0$ milligram of oxygen per liter.

Of the four options offered, only option C is correct. Option A results from failing to distinguish between the mean and median values in the data distribution. Option B results from assuming the median is 6, and option D results from assuming the median is 5.

SECTION IV

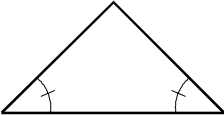

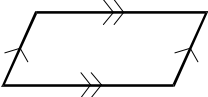
SAMPLE MULTIPLE-CHOICE ITEMS

This section presents some sample multiple-choice items for you to review as part of your preparation for the test. To demonstrate how each competency may be assessed, each sample item is accompanied by the competency number that it measures. While studying, you may wish to read the competency before and after you consider each sample item. Please note that the competency numbers will not appear on the actual test form.

An answer key follows the sample items. The answer key lists the item number and correct answer for each sample item. Please note that the answer key also lists the competency assessed by each item and that the sample items are not necessarily presented in competency order.

The sample items are included to illustrate the formats and types of items you will see on the test; however, your performance on the sample items should not be viewed as a predictor of your performance on the actual examination.

Definitions and Formulas

<p style="text-align: center;">PROBABILITY</p> <p>$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$</p> <p>$P(A \text{ and } B) = P(A) P(B A) = P(B) P(A B)$</p> <hr/> <p style="text-align: center;">GEOMETRY</p> <p style="text-align: center;">Congruent Angles</p>  <p style="text-align: center;">Congruent Sides</p>  <p style="text-align: center;">Parallel Sides</p>  <p style="margin-top: 20px;">Distance formula: $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$</p>	<p style="text-align: center;">ALGEBRA</p> <p>i $i^2 = -1$</p> <p>$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest where A is the final value P is the principal r is the interest rate t is the term n is divisions within the term</p> <p>$[x] = n$ Greatest integer function where n is the integer such that $n \leq x < n + 1$</p> <p>Quadratic formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</p> <hr/> <p style="text-align: center;">VOLUME</p> <p>Sphere: $\frac{4}{3}\pi r^3$</p> <p style="text-align: center;">AREA</p> <p>Rhombus: $\frac{1}{2} \text{diagonal}_1 \times \text{diagonal}_2$</p> <p>Circle: πr^2</p> <p>Lateral surface area of cylinder: $2\pi rh$</p> <p>Lateral surface area of cone: $\pi r \times (\text{slant height})$</p> <p>Surface area of sphere: $4\pi r^2$</p>
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TRIGONOMETRY

sine of $\angle A$: $\frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$

cosine of $\angle A$: $\frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$

tangent of $\angle A$: $\frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$

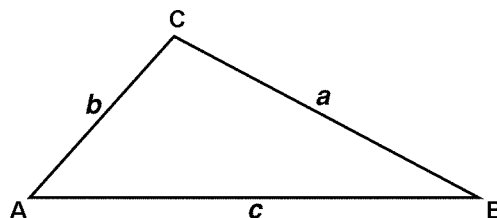
Pythagorean identity: $\sin^2 A + \cos^2 A = 1$

Law of sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

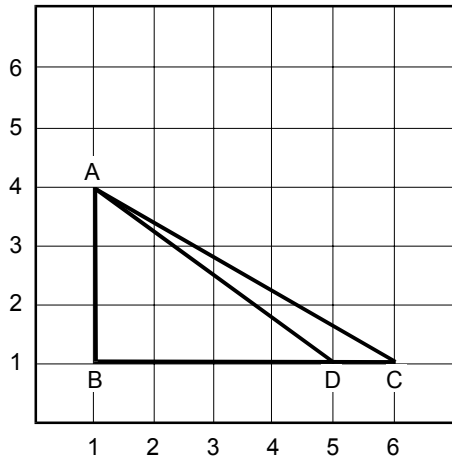
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$


END OF DEFINITIONS AND FORMULAS

Competency 001

1. Use the diagram below to answer the question that follows.



Which of the following measurements associated with the triangles in the diagram above can be expressed as a nonterminating, nonrepeating decimal?

- A. AD
- B. AC
- C. BC
- D. DC

Competency 001

2. A number set is dense if and only if for any elements a and b there exists an element x such that $a < x < b$. According to this definition, which of the following sets is dense?
- A. integers
 - B. natural numbers
 - C. rational numbers
 - D. complex numbers

Competency 002

3. If $\frac{a + 13b}{3a - b} = 3$, what is the value of $\frac{a^3}{b^3}$?
- A. $\left(-\frac{8}{5}\right)^3$
 - B. $\left(-\frac{7}{4}\right)^3$
 - C. 2^3
 - D. 7^3

Competency 002

4. A student is having difficulty with multiplication problems that involve a two-digit number times a one-digit number. Three examples of the student's work are shown below.

$\begin{array}{r} 2 \\ 27 \\ \times 4 \\ \hline 168 \end{array}$	$\begin{array}{r} 2 \\ 34 \\ \times 6 \\ \hline 304 \end{array}$	$\begin{array}{r} 1 \\ 43 \\ \times 5 \\ \hline 255 \end{array}$
--	--	--

Which of the following problems contains the same error as that made by the student in the three problems above?

A.

$$\begin{array}{r} 4 \\ 26 \\ \times 7 \\ \hline 422 \end{array}$$

B.

$$\begin{array}{r} 3 \\ 35 \\ \times 6 \\ \hline 540 \end{array}$$

C.

$$\begin{array}{r} 2 \\ 43 \\ \times 8 \\ \hline 324 \end{array}$$

D.

$$\begin{array}{r} 1 \\ 52 \\ \times 7 \\ \hline 704 \end{array}$$

Competency 003

5. A rectangular floor that measures 78 inches by 102 inches is to be completely covered with square tiles. The tiles are available in sizes with only whole-number side lengths. What is the smallest number of uncut tiles that could be used to cover the floor?
- A. 204
 - B. 221
 - C. 1,326
 - D. 3,978

Competency 003

6. On a trip from El Paso to Texarkana, a car is moving along a highway at a uniform rate of speed. At 9:00 A.M. it is $\frac{1}{4}$ of the way from El Paso to Texarkana. At 3:00 P.M. it is $\frac{7}{10}$ of the way from El Paso to Texarkana. Approximately what fraction of the way from El Paso to Texarkana was the car at 1:00 P.M.?
- A. $\frac{11}{20}$
 - B. $\frac{3}{5}$
 - C. $\frac{13}{20}$
 - D. $\frac{7}{10}$

Competency 004

7. On a recent homework assignment, a student wrote that -4^2 is equivalent to 16. A review of which of the following concepts would best address the student's error?
- A. meaning of exponents
 - B. order of operations
 - C. multiplication of negative numbers
 - D. distributive property

Competency 005

8. A student has been encountering some difficulty with subtraction problems involving three-digit numbers. Shown below are four examples of the student's work.

$\begin{array}{r} \overset{6}{6}\overset{1}{\cancel{7}}4 \\ - 259 \\ \hline 415 \end{array}$	$\begin{array}{r} \overset{4}{\cancel{5}}\overset{1}{8}6 \\ - 392 \\ \hline 194 \end{array}$	$\begin{array}{r} \overset{5}{\cancel{7}}\overset{1}{4}5 \\ - 486 \\ \hline 169 \end{array}$	$\begin{array}{r} \overset{2}{\cancel{4}}\overset{1}{3}4 \\ - 276 \\ \hline 68 \end{array}$
--	--	--	---

Which of the following would best address the student's errors?

- A. Have the student practice subtraction facts with flash cards.
- B. Use base-ten rods to demonstrate the standard algorithm to the student.
- C. Review subtraction problems involving two-digit numbers with the student.
- D. Use base-ten blocks to demonstrate to the student trading a ten for ones and trading a hundred for tens.

Competency 006

9. Define the function D such that $D(n)$ is the total number of factors of n . If the domain of D is the set of positive integers less than or equal to 15, what is the range of D ?
- A. $\{1, 3, 5\}$
- B. $\{1, 2, 3, 4, 6\}$
- C. $\{1, 2, 3, 4, 5, 6\}$
- D. $\{1, 2, 3, 5, 7, 11, 13\}$

Competency 006

10. Use the information below to answer the question that follows.

	Number of Vertices	Number of Lateral Faces	Number of Edges
Triangular Prism	6	3	9
Rectangular Prism	8	4	12
Hexagonal Prism	12	6	18
Octagonal Prism	16	8	24

If the total number of vertices, lateral faces, and edges of a prism is 288, how many sides does the base of the prism have?

- A. 42
- B. 44
- C. 46
- D. 48

Competency 007

11. The graph of the equation $4y = 3x + 4b$, where b is a constant, has an x -intercept of 3. What is the value of b ?

A. $-\frac{16}{9}$

B. -2

C. $-\frac{9}{4}$

D. -4

Competency 007

12. Given the graph of the equation $y = mx + b$, where $m < 0$, what will happen to the graph of the equation as the value of m increases while b remains fixed?

A. The graph will be translated in the positive x -direction.

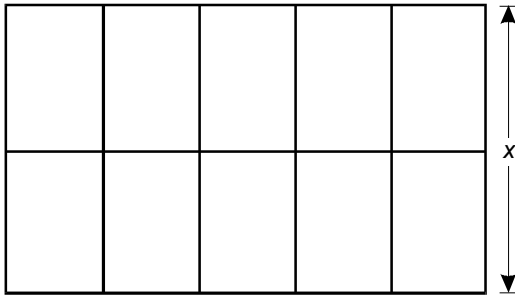
B. The graph will be translated in the positive y -direction.

C. The graph will be rotated counterclockwise about the origin.

D. The graph will be rotated counterclockwise about the y -intercept.

Competency 008

13. Use the diagram below to answer the question that follows.

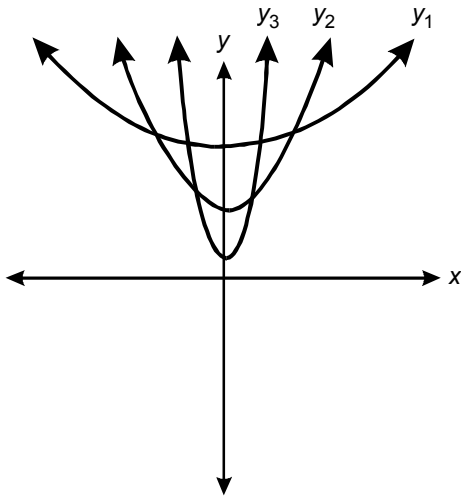


The diagram shows how a length of fence 180 feet long will be used to create 10 pens for holding animals. Which of the following equations represents the total area of the pens as a function of x ?

- A. $A(x) = 90x - x^2$
- B. $A(x) = 60x - 2x^2$
- C. $A(x) = 60x - 3x^2$
- D. $A(x) = 60x - 6x^2$

Competency 008

14. Use the diagram below to answer the question that follows.



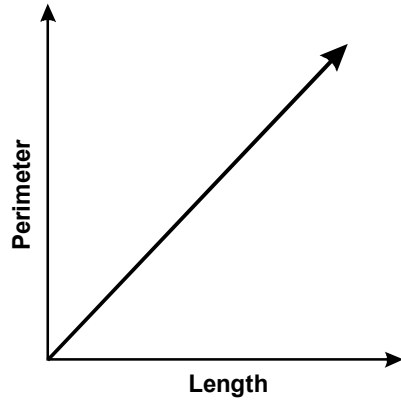
The diagram shows a family of functions in the form $y = ax^2 + bx + c$. Which of the following statements best describes the changes in the values of the coefficients as the graphs vary from y_1 to y_2 to y_3 ?

- A. a is increasing, $b = 0$, and c is increasing
- B. a is increasing, $b = 0$, and c is decreasing
- C. a is decreasing, b is increasing, and $c = 0$
- D. a is decreasing, b is decreasing, and $c = 0$

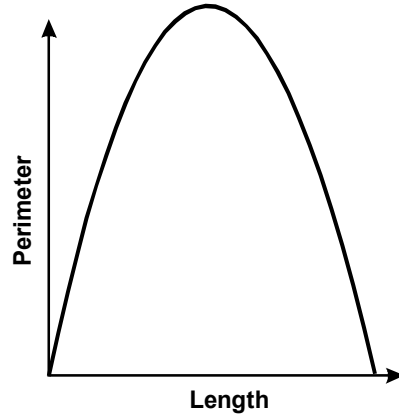
Competency 009

15. The area of a rectangle is constant. Which of the following represents the perimeter of the rectangle as a function of the length of one of its sides?

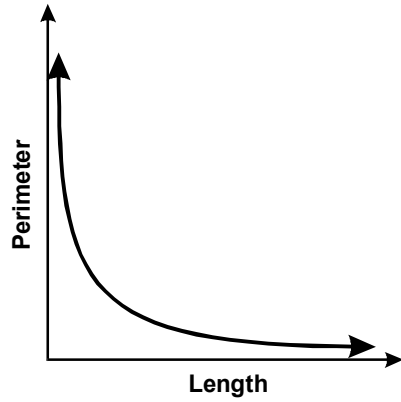
A.



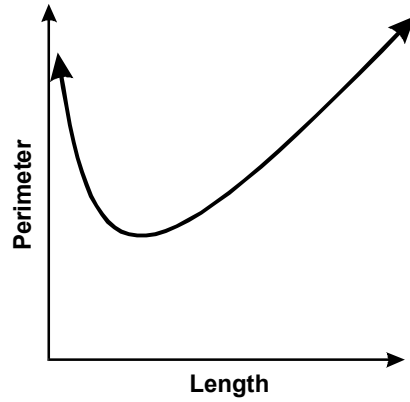
B.



C.

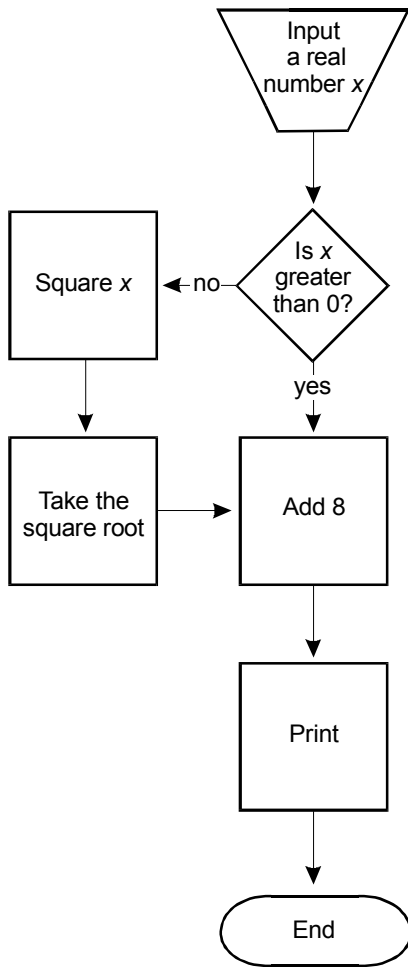


D.



Competency 009

16. Use the flowchart below to answer the question that follows.



What type of function does this flowchart represent?

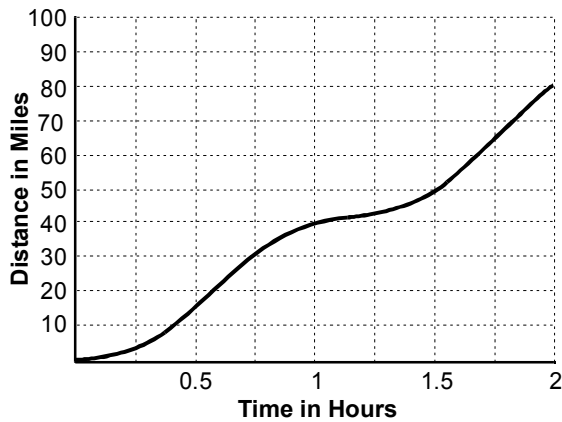
- A. linear
- B. radical
- C. quadratic
- D. absolute value

Competency 010

17. The methods of integral calculus can be used to solve which of the following problems?
- A. Find a formula for the distance between two points.
 - B. Find a formula for the area of a circle.
 - C. Find a formula for the size of an angle in a triangle.
 - D. Find a formula for the vertex of a parabola.

Competency 010

18. Use the graph below to answer the question that follows.



The graph above shows the distance traveled by a car over a two-hour period. What is the difference between the car's speed at $t = 1.75$ and the car's average speed during the second hour of the trip?

- A. -20 mph
- B. -10 mph
- C. 10 mph
- D. 20 mph

Competency 011

19. A student's solution to a homework problem involving systems of equations is shown below. A note from the student is included at the end of the problem.

Problem:	$4x - 2y = 8$ $-2x + 3y = 20$
Solution:	$4x - 2y = 8$ $-2y = -4x + 8$ $y = 2x - 4$ $4x - 2(2x - 4) = 8$ $4x - 4x + 8 = 8$ $8 = 8$

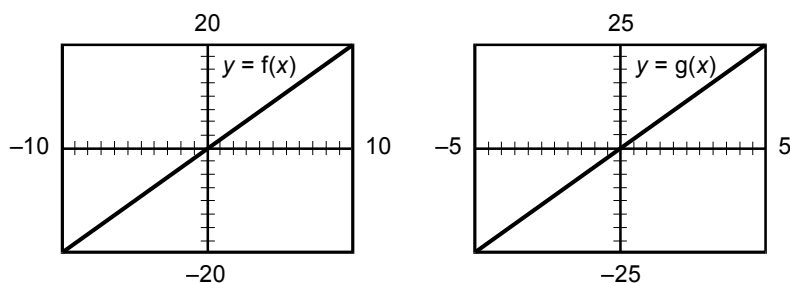
I'm not sure if I made an error. I solved the first equation for "y =", used substitution, and came up with the statement $8 = 8$. I checked over all of my arithmetic. Did I do anything wrong?

Which of the following responses would best address the student's note?

- A. There is nothing wrong with your approach. You've determined that the x value is 8. Now you just need to determine the y value.
- B. There is an error in your approach. You need to solve the first equation for " $x =$ " and then use substitution to determine the y value.
- C. There is nothing wrong with your approach. Your final statement of " $8 = 8$ " indicates that there are an infinite number of solutions to the problem.
- D. There is an error in your approach. After you solve the first equation for " $y =$ ", you need to substitute this expression for y into the other equation.

Competency 011

20. Use the graphs below to answer the question that follows.



A teacher asks the students in a class to use their graphing calculators to graph the functions $y = f(x)$ and $y = g(x)$, using the viewing windows shown above. This activity demonstrates that:

- A. lines with different y -intercepts can appear to have the same y -intercept when viewed in different windows.
- B. lines that appear to have the same slope in two viewing windows may have different slopes.
- C. lines that have the same slope in two different viewing windows are parallel.
- D. lines with the same y -intercept will appear equivalent when viewed in different windows.

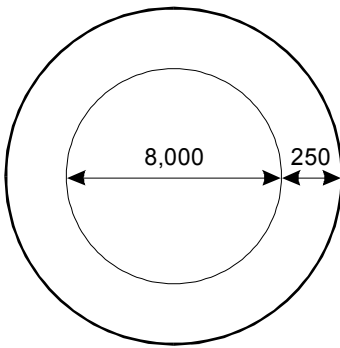
Competency 012

21. In class, students have been successful using algebra blocks to model addition. However, the teacher notices that in their homework problems, most of the students still make errors like $8x + 7y = 15xy$ and $4x + 9y = 13x$. Which of the following activities would be most helpful to these students?

- A. Do problems with drawings of rectangles and squares instead of xs and ys .
- B. Start with problems involving smaller numbers of xs and ys .
- C. Remind students of the associative, commutative, and distributive properties of numbers and operations.
- D. Reteach the lesson using blocks that differ in color instead of shape.

Competency 013

22. Use the diagram below to answer the question that follows.

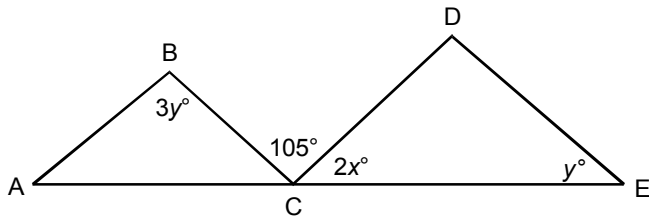


The orbit of a satellite is approximately 250 miles above the surface of the earth, which has a diameter of approximately 8,000 miles. If the satellite travels a total of about 4,000,000 miles in space during a 240-hour period, it will orbit the earth approximately once every:

- A. 90 minutes.
- B. 96 minutes.
- C. 180 minutes.
- D. 192 minutes.

Competency 014

23. Use the diagram below to answer the question that follows.

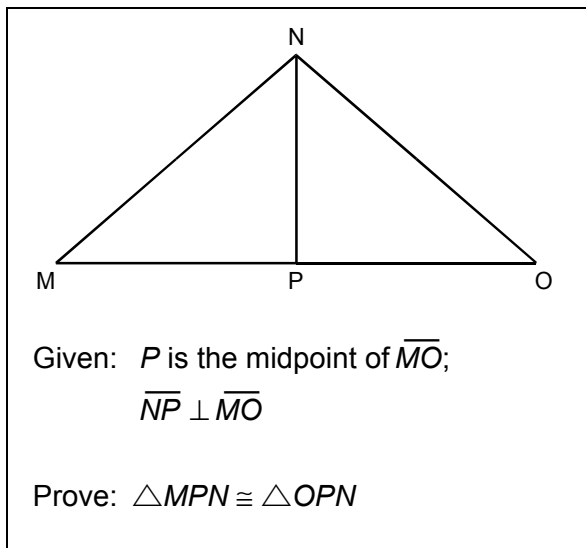


If \overline{AB} is parallel to \overline{CD} and \overline{BC} is parallel to \overline{DE} , what is the value of $x - y$?

- A. -15
- B. $-16\frac{1}{4}$
- C. 15
- D. $17\frac{1}{2}$

Competency 014

24. Use the diagram and information below to answer the question that follows.



Which of the following triangle relations is necessary to complete the proof?

- A. SAS
- B. SSA
- C. SSS
- D. ASA

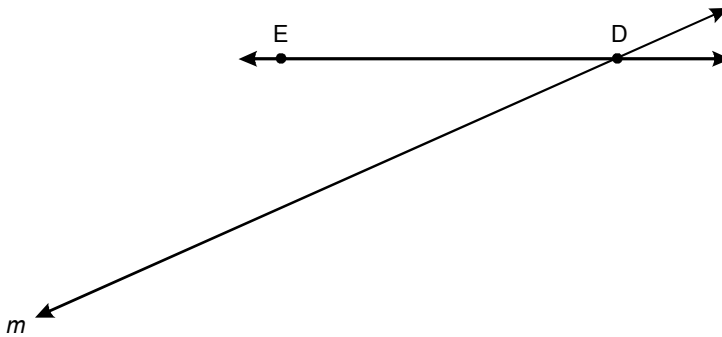
Competency 015

25. A hollow right circular cylinder with open ends has a radius of r and a height of $2r$. The cylinder is cut along one side and unrolled to form a rectangle. What are the dimensions of the rectangle?

- A. πr and $2r$
- B. $2\pi r$ and $2r$
- C. πr and $2\pi r$
- D. $2\pi r$ and $4\pi r$

Competency 016

26. Use the diagram below to answer the question that follows.

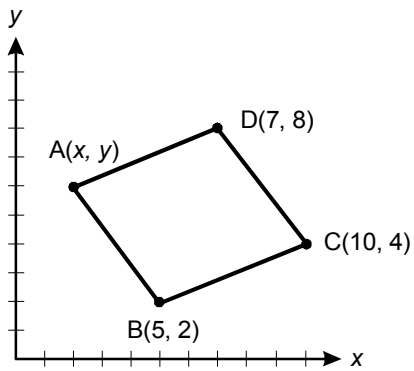


In the diagram above, point G is located below line m such that line m is an axis of reflective symmetry of $\triangle DEG$. Point D is then reflected over \overline{EG} to form point F . Which of the following is the most accurate description of quadrilateral $DEFG$?

- A. rhombus
- B. square
- C. parallelogram
- D. rectangle

Competency 016

27. Use the diagram below to answer the question that follows.

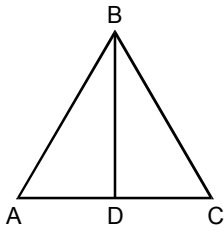


Quadrilateral $ABCD$ is a parallelogram. What is an equation of line AB ?

- A. $3y - 4x = -23$
- B. $3y - 4x = -14$
- C. $3y + 4x = 23$
- D. $3y + 4x = 26$

Competency 017

28. Use the figure and information below to answer the question that follows.



In the figure of $\triangle ABC$ above, $AB = BC$ and \overline{BD} bisects $\angle B$. Students are asked to prove that $\triangle ADB \cong \triangle CDB$. One student submits the following proof.

Statements	Reasons
1. $AB = BC$	1. Given
2. $AD = DC$	2. Definition of a bisector
3. $BD = BD$	3. Reflexive property
4. $\triangle ADB \cong \triangle CDB$	4. SSS

Which of the following best describes the error made by the student?

- A. Reason 1 is an assumption and not a given.
- B. Statement 2 assumes that the line segment is bisected.
- C. Reason 3 should read *Symmetric property*.
- D. Statement 4 does not follow from the previous statements.

Competency 017

29. A teacher asks students to separate cardboard models of several different polygons into two groups depending on whether the polygons are concave or convex. Several students are having difficulty distinguishing the difference between these types of polygons. Which of the following activities would most likely help the students understand the difference between these types of polygons?
- A. Have the students measure the interior angles of the polygons to determine if they are equal.
 - B. Have the students wrap a string around the polygons and describe their observations.
 - C. Have the students trace the polygons on grid paper and count the number of squares inside each polygon.
 - D. Have the students try to fold the polygons in half so that the two halves match each other without overlapping.

Competency 018

30. A teacher gives a student the following problem.

One can of Chewy Nuts holds 2 pounds 5 ounces of nuts. A case of Chewy Nuts contains a dozen cans. How many pounds of nuts are in a case of Chewy Nuts?

The student answers that a case contains 30 pounds of nuts. Which of the following questions should the teacher ask first in order to help the student understand his or her mistake?

- A. What is 5 times 12?
- B. How many units are in a dozen?
- C. What is the remainder of $60 \div 16$?
- D. How many ounces are in a pound?

Competency 019

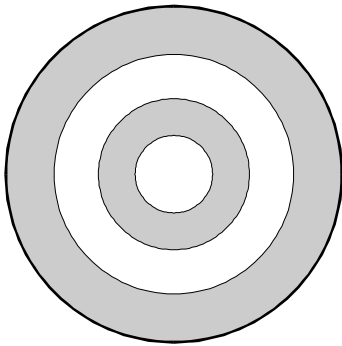
31. An elementary school is divided into three buildings: Building A, Building B, and Building C. In the school, 40% of the students are in Building A and 32% of the students are in Building B. When these data are presented in a circle graph, what is the measure of the central angle representing the number of students in Building C?
- A. 28°
 - B. 72°
 - C. 100.8°
 - D. 129.6°

Competency 020

32. Two tickets are drawn with replacement from a box containing four tickets numbered 1 through 4. What is the probability that the product of the numbers on the two tickets is 9 or greater?
- A. $\frac{1}{4}$
 - B. $\frac{1}{3}$
 - C. $\frac{1}{2}$
 - D. $\frac{3}{4}$

Competency 020

33. Use the diagram below to answer the question that follows.



Beginning at the center, the concentric circles on the target have radii of 1, 2, 3, and 4 inches, respectively. If a dart thrown at the target has an equal probability of hitting any spot on the target, what is the probability of a dart hitting a shaded area?

- A. $\frac{5}{8}$
- B. $\frac{11}{16}$
- C. $\frac{3}{4}$
- D. $\frac{13}{16}$

Competency 021

34. A survey based on a random sample of 200 male giraffes evaluates a 95% confidence interval for the giraffes' average height to be 17' 10" \pm 3". This statistic implies that:
- A. 95% of all giraffes are between 17' 7" and 18' 1" in height.
 - B. 95% of all male giraffes are between 17' 7" and 18' 1" in height.
 - C. there is a 95% chance that the interval between 17' 7" and 18' 1" includes the true average height of all giraffes.
 - D. there is a 95% chance that the interval between 17' 7" and 18' 1" includes the true average height of all male giraffes.

Competency 021

35. A teacher gives some students a coin and asks them to test it for balance. They flip the coin 10 times with the following results: HHHHHHHHHH. After which flip can the students be 95% certain that the coin is not balanced, i.e., heads (H) are more likely to occur than tails (T)?
- A. 5th
 - B. 6th
 - C. 7th
 - D. 8th

Competency 022

36. Students in a fourth-grade class are engaged in a partner activity in which they draw a marble from a bag with five red marbles and five blue marbles, replace the marble, shake the bag, and draw again. The teacher overhears a student telling his partner that since they've drawn six red marbles in a row, they're likely to get a blue marble on the next draw. This student would most benefit from instruction designed to promote understanding of which of the following concepts?
- A. addition and multiplication properties of probability
 - B. sample space
 - C. dependent and independent events
 - D. complementary events

Competency 022

37. Students are learning in their science class that biological diversity results from a genetic code that involves the arrangement of four different molecules into specific sequences. This topic would provide an opportunity in a mathematics class to discuss which of the following topics?
- A. probability
 - B. exponential functions
 - C. geometric progressions
 - D. combinations and permutations

Competency 023

38. A student determines that there is a one-in-four chance that three consecutive coin flips will all come up the same (i.e., all heads or all tails). The student then conducts eight separate experimental trials of three coin flips each. However, none of the eight trials results in all heads or all tails. To help the student reconcile her calculation with the experimental results, the teacher should instruct the student to:
- A. perform a much larger number of experimental trials.
 - B. consider whether she should have added probabilities, rather than multiplying.
 - C. use a tree diagram to check her calculation.
 - D. try a different coin in case the coin she is using is biased.

Competency 024

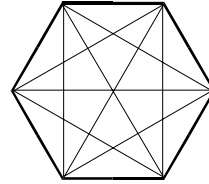
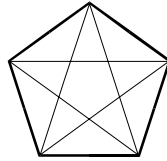
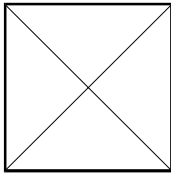
39. Which of the following is an example of inductive reasoning?
- A. Before determining the average of a set of data, a student recalls the distinction between mean, median, and mode.
 - B. After learning that a square is a rectangle and a rectangle is a parallelogram, a student concludes that a square is a parallelogram.
 - C. To determine whether 37 is a prime number, a student lists each of the smaller whole numbers and considers whether any of them are factors of 37.
 - D. After multiplying several pairs of whole numbers, a student forms the hypothesis that only two odd numbers can have a product that is odd.

Competency 024

40. An object, placed in a tank of water for 24 hours, has a temperature modeled by $H(t) = t(t + 2)(t - 15)(t - 30)$, where t is the number of hours in the tank. At what times, t , is the temperature of the object equal to zero?
- A. $-2, 0, 15,$ and 30
 - B. $0, 15,$ and 30
 - C. 15 and 30
 - D. 0 and 15

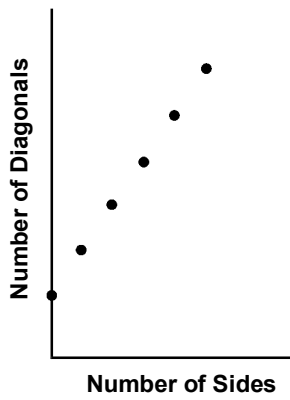
Competency 025

41. Use the diagrams below to answer the question that follows.

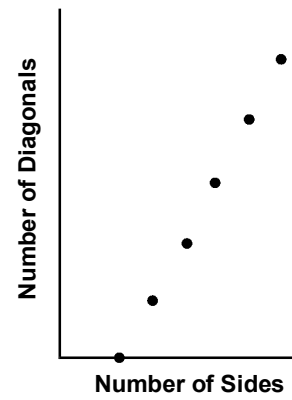


The diagrams above show several regular polygons with all possible diagonals drawn. Which of the following graphs best represents the relationship of the number of sides of a polygon to the number of diagonals?

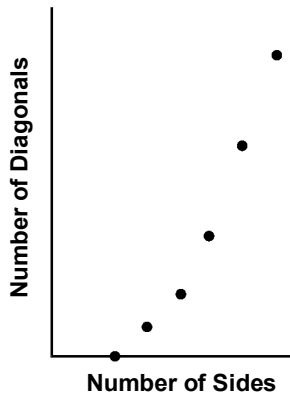
A.



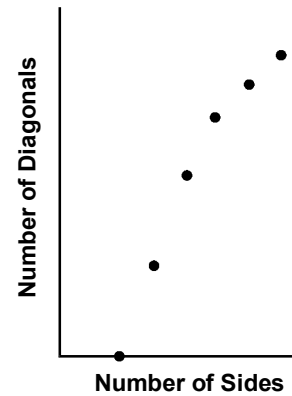
B.



C.



D.



Competency 025

42. The loudness of a sound, or sound intensity, (in watts per square meter) is inversely proportional to the square of the listener's distance from the source of the sound. At a distance of two meters, the sound intensity of a machine is 8.0×10^{-4} . What is the intensity at a distance of 10 meters from the machine?
- A. 3.2×10^{-5}
 - B. 4.0×10^{-4}
 - C. 1.6×10^{-3}
 - D. 2.8×10^{-2}

Competency 026

43. In addition to his or her own classroom teaching duties, a Master Mathematics Teacher could reasonably expect to become involved in:
- A. performing regular evaluations of other mathematics teachers in the school.
 - B. regularly co-teaching with a classroom teacher who is experiencing difficulty implementing the mathematics curriculum.
 - C. diagnosing and working with small groups of students with disabilities.
 - D. collaborating with classroom teachers individually and collectively in improving the quality of mathematics instruction in the school.

Competency 026

44. A faculty committee led by a school's Master Mathematics Teacher has recommended that the school purchase a group license for a mathematics software package. The Master Mathematics Teacher is most likely to gain administrative support for this purchase by:
- A. demonstrating the features and capabilities of the software to the administration.
 - B. describing how the software will help mathematics teachers achieve the school's instructional goals.
 - C. documenting the use of similar software at schools in surrounding districts.
 - D. asking faculty and parents/guardians to voice their support for use of the software.

Competency 027

45. A middle school curriculum committee has decided that science and mathematics teachers should work together to develop a series of teaching units that integrate the learning of the two subjects. Which of the following describes the most effective and appropriate role for the Master Mathematics Teacher to take in this project?
- A. Collaborate with the science teachers to determine the mathematics knowledge required for the science instruction and then teach the mathematics portions of the science classes.
 - B. Provide the science teachers with copies of the mathematics curriculum and assist them in designing science instruction that engages suitable mathematics learning.
 - C. Collaborate with science and mathematics teachers to communicate mathematics instruction needs and coordinate mathematics teaching and science teaching schedules.
 - D. Instruct science teachers in the mathematical functions necessary for the learning of science and be available to coach students as needed during science classes.

Competency 027

46. The Master Mathematics Teacher is observing a new mathematics teacher, Mr. Sanborn. Although Mr. Sanborn's students appear to understand the concepts being taught, the Master Mathematics Teacher is concerned that they may lack the basic skills necessary to perform according to district curriculum standards. Which of the following would be the most appropriate way for the Master Mathematics Teacher to address his concerns?
- A. Acknowledge Mr. Sanborn's effective teaching strategy and also encourage him to assess his students' learning in terms of curriculum-based computation standards.
 - B. Point out that Mr. Sanborn's lessons do not specifically address curriculum standards and explain that he will need to revise lessons to spend more time teaching basic computation skills.
 - C. Discuss the omission with Mr. Sanborn and offer to provide him with lesson plans to acquaint him with what he needs to fill in the gap in his instruction.
 - D. Bring Mr. Sanborn's style to the attention of the head of the mathematics department and suggest that workshops on teaching computation skills be made a priority for professional development.

ANSWER KEY

Item Number	Correct Answer	Competency
1	B	001
2	C	001
3	C	002
4	A	002
5	B	003
6	A	003
7	B	004
8	D	005
9	B	006
10	D	006
11	C	007
12	D	007
13	B	008
14	B	008
15	D	009
16	D	009
17	B	010
18	D	010
19	D	011
20	B	011
21	A	012
22	B	013
23	A	014

Item Number	Correct Answer	Competency
24	A	014
25	B	015
26	A	016
27	D	016
28	B	017
29	B	017
30	D	018
31	C	019
32	A	020
33	A	020
34	D	021
35	A	021
36	C	022
37	D	022
38	A	023
39	D	024
40	D	024
41	C	025
42	A	025
43	D	026
44	B	026
45	C	027
46	A	027

SECTION V

CASE STUDY ASSIGNMENT

In addition to the multiple-choice section, the Master Mathematics Teacher (MMT) test will include one case study assignment that requires a written response. The written-response score will be combined with the multiple-choice score to produce a total test scaled score.

Included in this section is a description of the case study assignment, an explanation of the way case study assignment responses will be scored, and one sample case study assignment.

How Case Study Assignment Responses Are Scored

Responses will be scored on a four-point scale (see next page). Each point on the scale represents the degree to which the performance characteristics (see below) are demonstrated in the response.

The score point descriptions reflect typical responses at each score point. Although the score assigned corresponds to one of the score points, individual responses may include attributes of more than one score point.

PERFORMANCE CHARACTERISTICS

PURPOSE	The extent to which the candidate responds to the components of the assignment in relation to relevant competencies in the Master Mathematics Teacher 4–8 test framework.
APPLICATION OF KNOWLEDGE	Accuracy and effectiveness in the application of knowledge as described in relevant competencies in the Master Mathematics Teacher 4–8 test framework.
SUPPORT	Quality and relevance of supporting details in relation to relevant competencies in the Master Mathematics Teacher 4–8 test framework.
RATIONALE	Soundness of reasoning and depth of understanding of the assigned task in relation to relevant competencies in the Master Mathematics Teacher 4–8 test framework.
SYNTHESIS	The extent to which the candidate is able to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 4–8 in an applied context.

SCORE SCALE

Score	Score Point Description
4	<p>The "4" response reflects thorough knowledge and understanding of relevant competencies in the Master Mathematics Teacher 4–8 test framework.</p> <ul style="list-style-type: none"> • The response addresses all components of the assignment and fully completes the assigned task. • The response demonstrates an accurate and very effective application of relevant knowledge. • The response provides strong supporting evidence with specific and relevant examples. • The response demonstrates clear, logical reasoning and a comprehensive understanding of the assigned task. • The response demonstrates strong ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 4–8.
3	<p>The "3" response reflects sufficient knowledge and understanding of relevant competencies in the Master Mathematics Teacher 4–8 test framework.</p> <ul style="list-style-type: none"> • The response addresses most or all components of the assignment and sufficiently completes the assigned task. • The response demonstrates a generally accurate and effective application of relevant knowledge; minor problems in accuracy or effectiveness may be evident. • The response provides sufficient supporting evidence with mostly specific and relevant examples. • The response demonstrates sufficient reasoning and an overall understanding of the assigned task. • The response demonstrates sufficient ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 4–8.
2	<p>The "2" response reflects partial knowledge and understanding of relevant competencies in the Master Mathematics Teacher 4–8 test framework.</p> <ul style="list-style-type: none"> • The response addresses at least some components of the assignment and/or partially completes the assigned task. • The response demonstrates a partial and/or ineffective application of relevant knowledge; significant inaccuracies may be evident. • The response provides minimal supporting evidence with few relevant examples; some extraneous or unrelated information may be evident. • The response demonstrates limited reasoning and understanding of the assigned task. • The response demonstrates partial ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 4–8.
1	<p>The "1" response reflects little or no knowledge or understanding of relevant competencies in the Master Mathematics Teacher 4–8 test framework.</p> <ul style="list-style-type: none"> • The response addresses few components of the assignment and/or fails to complete the assigned task. • The response demonstrates a largely inaccurate and/or ineffective application of relevant knowledge. • The response provides little or no supporting evidence, few or no relevant examples, or many examples of extraneous or unrelated information. • The response demonstrates little or no reasoning or understanding of the assigned task. • The response demonstrates little or no ability to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 4–8.
U	<p>The "U" (Unscorable) will be assigned to responses that are off topic/off task, illegible, primarily in a language other than English, or are too short or do not contain a sufficient amount of original work to score.</p>
B	<p>The "B" (Blank) will be assigned to written response booklets that are completely blank.</p>

Note: Your written response should be your original work, written in your own words, and not copied or paraphrased from some other work.

Scoring Process

Case study assignment responses are scored on a scale of 1 to 4. Each response is evaluated by a minimum of two scorers with expertise in mathematics instruction. All scorers have successfully completed standardized orientation and are calibrated to the scoring criteria throughout the scoring session.

Analytic Notation

Examinees who do not pass the test and do not perform satisfactorily on the case study assignment will receive information concerning specific aspects of the written response that show a need for improvement. This information will be provided for examinees to use in preparing to retake the test.

If you do not pass the test or perform satisfactorily on the case study assignment, your score report will indicate one or more of the following areas for improvement in your written response. These areas are based on the performance characteristics in the score scale.

- Purpose
- Application of Knowledge
- Support
- Rationale
- Synthesis

Preparing for the Case Study Assignment

Following is one sample case study assignment that represents the type of question you will see on the MMT test.

In preparing for the case study assignment component of the test, you may wish to draft a response to the question by reading the case study and planning, writing, and revising your essay. You should plan to use about 90 minutes to respond to the sample case study assignment. Also, since no reference materials will be available during the test, it is recommended that you refrain from using a dictionary, a thesaurus, or textbooks while writing your practice response.

After you have written your practice response, review your response in light of the score point descriptions. You may also wish to review your response and the score scale with staff in your MMT preparation program.

General Directions for Responding to the Case Study Assignment

DIRECTIONS FOR CASE STUDY ASSIGNMENT Master Mathematics Teacher 4–8

General Directions:

This section of the test consists of one case study assignment. For this assignment, you are to prepare a written response and record it in the area provided in the written response booklet.

Read the case study assignment carefully before you begin to write. Think about how you will organize what you plan to write. You may use any blank space provided in this test booklet to make notes, create an outline, or otherwise prepare your response. ***Your final response, however, must be written in the written response booklet.***

Evaluation Criteria:

Your written response will be evaluated based on the extent to which it demonstrates the knowledge and skills required to perform the roles of the Master Mathematics Teacher 4–8. You may draw from research and your professional experience. (Citing specific research is not required.)

Read the assignment carefully to ensure that you address all components. Your response to the assignment will be evaluated based on the following criteria:

- **PURPOSE:** The extent to which you respond to the components of the assignment in relation to relevant competencies in the Master Mathematics Teacher 4–8 test framework.
- **APPLICATION OF KNOWLEDGE:** Accuracy and effectiveness in the application of knowledge as described in relevant competencies in the Master Mathematics Teacher 4–8 test framework.
- **SUPPORT:** Quality and relevance of supporting details in relation to relevant competencies in the Master Mathematics Teacher 4–8 test framework.
- **RATIONALE:** Soundness of reasoning and depth of understanding of the assigned task in relation to relevant competencies in the Master Mathematics Teacher 4–8 test framework.
- **SYNTHESIS:** The extent to which you are able to synthesize the knowledge and skills required to perform the multifaceted role of the Master Mathematics Teacher 4–8 in an applied context.

The assignment is intended to assess knowledge and skills required to perform the roles of the Master Mathematics Teacher 4–8, not writing ability. Your response, however, must be communicated clearly enough to permit a valid judgment about your knowledge and skills. Your response should be written for an audience of educators knowledgeable about the roles of the Master Mathematics Teacher 4–8.

The final version of your response should conform to the conventions of edited American English. Your response should be your original work, written in your own words, and not copied or paraphrased from some other work. You may, however, use citations when appropriate.

Sample Case Study Assignment

47. **Classroom Context:** This case study focuses on a seventh-grade mathematics teacher, Ms. Conway, who is teaching her second-period class an introductory lesson on percentages and how to solve percentage equations and problems. The class, which meets for 50 minutes a day, is composed of students who achieve at various levels.

Master Mathematics Teacher Task: Last year, Ms. Conway's students performed poorly on the classroom tests and statewide assessments on percentages. Ms. Conway has asked the Master Mathematics Teacher (MMT) to observe her class and provide assistance as she works on teaching percentages. The MMT has agreed to observe her lesson. Ms. Conway shows the MMT a lesson plan that she intends to use on the day of the MMT's observation. On the following pages, you will find:

- information from Ms. Conway regarding her previous instruction for this class;
- the lesson plan implemented on the day of the MMT's observation;
- an assignment given by Ms. Conway to her class;
- excerpts of notes taken by the MMT while observing Ms. Conway's lesson; and
- representative samples of student work from the class.

Using these materials, write a response in which you apply your knowledge of mathematics, mathematics instruction, and mentoring to analyze this case study. Your response should include the following information:

- An analysis of two significant weaknesses in the effectiveness of the lesson on percentages. Cite evidence from the case study to support your observations.
- A full description of two instructional strategies or assignments that would be effective for Ms. Conway to use to address the weaknesses you have identified. Be sure to describe one strategy or assignment for each of the weaknesses you identified.
- An explanation of why each of the strategies or assignments you have described would be effective in improving Ms. Conway's instruction on percentages.
- A full description of two appropriate actions you would take as a mentor teacher to help Ms. Conway implement the strategies or assignments you have described.

Information from the teacher regarding previous instruction: The students in this class have completed units on fractions, ratios, proportions, and decimals. The teacher feels that they understand these topics both numerically and graphically, recognize the relationships among them, and are able to solve word problems involving these concepts. The students have also completed a unit on solving equations of the form $ax = b$.

LESSON PLAN

Objective: Solve percentage equations and use percentages to solve real-life problems.

Warm-Up

- Write each of the following fractions as a decimal.
 1) $\frac{50}{100}$ 2) $\frac{75}{100}$ 3) $\frac{5}{100}$ 4) $\frac{24}{100}$ 5) $\frac{136}{100}$

Presentation of Material

- Explain how to convert a decimal to a percentage by moving the decimal point.
- Have students convert the decimals found above into percentages.
- Discuss three basic types of percentage problems:

<i>Question</i>	<i>Given</i>	<i>Find</i>
1) a is what percent of b ?	a and b	p
2) What is p percent of b ?	p and b	a
3) a is p percent of what?	a and p	b

- Explain that each percentage problem can be solved by substituting the two given quantities into the statement $p\%$ of b is a , and that this statement can be written as the equation $p \cdot b = a$. Mention that $p\%$ must be expressed in decimal form before solving the equation.
- Complete the following examples with the entire class:
 - What is 33% of 180?
 - 72 is what percent of 90?
 - 6 is 25% of what number?
 - A \$15 book is discounted 20%. What is the amount of the discount?

Classwork

- In groups of two, students work on the assignment "Solving Percentage Equations."*

Homework

- Finish "Solving Percentage Equations" independently at home.

*A copy of the assignment follows this lesson plan.

ASSIGNMENT

Solving Percentage Equations

For each problem below, do the following:

- Translate the problem into the statement $p\%$ of b is a .
- Write the statement as an equation.
- Solve the equation.

Show all of your work.

1) What is 2% of 360?

2) 20 is what percent of 50?

3) 25% of 270 is what?

4) 60 is 120% of what number?

5) What distance is 6% of 100 yards?

6) 3 dollars is 2% of what amount?

7) 9 people is what percent of 40 people?

8) 28% of what amount is \$240?

SELECTED EXCERPTS FROM THE MMT'S OBSERVATION NOTES

- *Students do the warm-up problems individually at the beginning of class. Few students encounter problems.*
- *Ms. C states the goal for the day is for students to understand how to solve equations involving percentages and how to use percentages to solve problems.*
- *Ms. C tells students that decimals can be changed to percentages by moving the decimal point two places to the right. Students convert the decimals found in the warm-up exercises into percentages with little difficulty.*
- *Three basic types of questions that can be asked are written on the board.*
- *Students are told that the statement " $p\%$ of b is a " can be used to solve any of the three questions, and this statement is equivalent to the equation $p \cdot b = a$, where p is written in decimal form.*
- *Ms. C works through four basic examples of solving percentage equations.*
 - *While doing a problem involving the value 33%, a student asks, "What does 33% mean?" Ms. C responds that "the word percentage means 'out of one hundred,' therefore 33% means 33 out of one hundred, and you know how to convert that to a decimal."*
 - *Many students can solve problems of the form "what is $x\%$ of y ?" but almost all have difficulty with problems of the form " x is $y\%$ of what number?"*
 - *While doing a problem involving the value 25%, a student asks, "Isn't 25% the same as one-quarter?" Ms. C responds that it is.*

SELECTED EXCERPTS FROM THE MMT'S OBSERVATION NOTES (continued)

- *Students work in pairs on the assignment on solving percentage problems. The teacher walks around the classroom and responds to individual questions as they arise.*
 - *Many students have difficulty with problems of the form "x is y% of what number?" and "x is what % of y?"*
 - *A few more questions arise about the meaning of percentage.*
 - *One pair of students asks whether 25% can be shown "with a picture." The response is "that will be done in the next class period."*
- *Class ends and the students are asked to complete the remainder of the assignment for homework.*

**SAMPLE STUDENT WORK FROM "SOLVING PERCENTAGE EQUATIONS"
ASSIGNMENT**

The problems below are representative samples of student work from the class.

Student J.K.

1). 2% of $b = 360$
 $20b = 360$
 $b = 18$

2). 20 percent of $b = 50$
 $20b = 50$
 $b = \frac{50}{20} = \frac{5}{2} = 2.5$
 which is 25%

3). 25% of 270
 $.25b = 270$
 $b = 1080$

4). 120% of 60 = a
 $1.2(60) = a$
 $a = 72$

5). 6% of 100 = a
 $.06(100) = a$
 $a = 6$

6). 2% of 3 = a
 $.02(3) = .06$
 $a = .06$

7). 9p% = 40
 ?

8). 28% of $b = 240$
 $28b = 240$
 $b = 8.57$

Student M. N.

1). $p\%$ of $b = a$
 2% is 2 out of 100 = $\frac{2}{100}$
 $\frac{2}{100}$ of 360 = $\frac{2}{100}(360) = 2(3.6) = 7.2$

2.) 20 is what percent of 50?
 need to find $p\%$
 $p\%$ of $b = a$

$p\%20 = 50$

$p\% = \frac{50}{20} = \frac{5}{2}$

$p\% = 2.5 = 250\%$ OR 0.025% ???

no way!

try

$p\%50 = 20$

$p\% = \frac{20}{50} = \frac{40}{100} = 40\%$

3). 75% of 270 is the same as
 $\frac{3}{4}(270) = 67.5$

4). 120% of 60 = a ???
 or 120% of $b = 60$???

5). 6% OF 100 yards = $0.06(100) = 6$
 yards

6). 3 is 2% of a or 2% of $b = 3$
 not sure

7). 9 is what percent of 40?
 $p\%$ of 40 = 9
 $p\% = 9/40 = .225 = 22.5\%$

8). 28% of $b = 240$
 stuck

SECTION VI

PREPARATION RESOURCES

The resources listed below may help you prepare for the TExMaT test in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

Journals

Mathematics Teacher, National Council of Teachers of Mathematics.

Mathematics Teaching in the Middle School, National Council of Teachers of Mathematics.

Middle Ground, National Middle School Association.

Middle School Journal, National Middle School Association.

Other Sources

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Online Resources

Mathematics TEKS Toolkit, <http://www.tenet.edu/teks/math>

National Council of Teachers of Mathematics, <http://www.nctm.org>

Texas Education Agency—Math Initiative, <http://www.tea.state.tx.us/math>

